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ON THE CHOICE OF NUMERICAL METHODS FOR SOLVING DYNAMIC EQUATIONS FOR CONTROL SYSTEMS WITH EMBEDDED CALCULATING TOOLS

Approaches to choice of optimum method as for accuracy and amount of computation at the integration step that makes it possible to range numerical implementation methods for different levels of acceptable error based on evaluation of optimality criterion.

Key words: *choice of numerical method, solving dynamic equations, algorithms for embedded calculating tools.*

Introduction. While designing fast high-precision controlled electromechanical systems of mechatronic type, one is facing the problem of calculation and control subsystem synthesis, characteristic feature of which is requirement of real-time operation with limited computational resources. In order to improve dynamic characteristics of electromechanical systems, modern control subsystems often use model of the controlled object. Therefore, to obtain desired characteristics of the subsystem as for operation speed and accuracy at the design stage, we should solve the problem of algorithm support that especially concerns the choice of an effective algorithm for numerical integration of dynamic equations for the controlled object, while this procedure appears to be a significant part of overall computational work [1–3]. The use of structurally-oriented approach to modelling complex electromechanical systems also requires effective method of mathematical description and numerical implementation of the system's structural elements. Since modelling process involves performance of an action sequence, namely, choice of mathematical description method for the modeled object, choice of an appropriate computational method, computational algorithm, appropriate program modules, then while creating such action sequence it is helpful to use an integrated approach. The most exhaustive study of this process includes such stages as analysis of the modelling problem, theoretical (analytical) study of computational methods and computational algorithms, and experimental study of computational algorithms.

Analysis of the problems involves identifying a priori information, determining characteristics and quantitative indicators that allow for their systematization and selection of the most effective computational method from those available in each particular case. In order to ensure efficient computer implementation of models, it is necessary to optimize computation [4], while searching for an optimum algorithm under some criteria: from the theoretical perspective, considering the class of problems and set of algorithms, and from the practical point of view, through selection of the best algorithm by comparing relevant characteristics of existing algorithms for solution of a given class of problems. It is therefore advisable to define the most common functional characteristics of the problems and perform their systematization accordingly. More detailed systematization should be done yet within each class, while specifying subclasses of problems by available a priori information. Classification of computational methods can be performed in a variety of ways: by criterion of presenting initial data and solution results (analytical and numerical), by functional and qualitative characteristics, by discretization method, by particularities of computer tools used etc. The choice of computational methods can be performed through developing of certain selection criteria based on requirements of the modelling task. Let's exemplify it by selection of numerical integration method for dynamic equations.

The best method selection problem. Traditionally equations of controlled motion dynamic of object can be represented as Cauchy problem for system of ordinary differential equations:

$$y' = f\left(y, U, p\right),\ y\left(0\right) = y_0, \qquad (1)$$

where y — vector of state variables, components of which in the general case are output coordinates of dynamic system (deviations of controlling elements, different replaceable parameters, etc.); U — vector of control signals that act at the input of the object control subsystem; y_0 — vector of initial values of state variables; p — vector of parameters that characterize properties of the object and its control subsystem, as well as properties of the external environment.

It should be noted that often among equations of the system (1) particular equation groups can be identified, integration of which is quite simple or is perhaps possible in an explicit form. Such equations include, for example, a group of one-dimensional equations that bind deviation values of control elements with values of appropriate control signals. The above equations are relatively simple, and the issue of their numerical solution is quite fully developed. However, solution of equations presented by the system (1) is the most complex and specific problem for given type of dynamic objects. The choice of numerical method is of particular importance just for solution of such equations.

Optimum method for computational solution of Cauchy problem in the above case means a method that provides the fastest approximate solution of the object dynamic equations, provided that error of the approximate solution does not exceed the preset value.

Complexity of the best method choice problem for computational solution of differential equations is a consequence of the fact that error of approximate solution depends heavily on the type of the equation system. It is reasonable to keep in mind that mathematical models of the object dynamics, in fact, differ only by value of the parameters p included in (1), as these equations in all cases follow the laws of conservation of momentum. However, even in this case the choice of «the best» method is a complex task that is closely related to the problem of a priori or a posteriori obtainment of precision characteristics of considerable number of methods that can be applied. Solving the problem of the choice of «the best» numerical method depends on determination of the optimality criterion. Typically, the optimality criterion is a composite function of certain characteristics (criterion arguments) of elements of the set, in which the optimum element is sought for. In this case, such set is a set of numerical methods for solving of differential equations. Optimum element refers to an element of the set, on characteristics of which the optimality criterion has global extreme value. When choosing «the best» method for numerical integration of differential equations, it is desirable to include in the optimality criterion arguments the following: characteristics of accuracy of the method, amount of computation at the integration step and the step size. These characteristics adequately reflect the properties of numerical methods, consideration of which is necessary for efficient algorithmic support [4–6].

To find the method that requires minimum operation speed and has an error that does not exceed preset allowable value δ_∂, the minimum value selection criterion is used

$$S_i = \frac{N_i}{\delta_i^{-1}\left(\delta_\partial\right)}, \tag{2}$$

where i — method index, N_i — amount of computation at the step of integration by the i^{th} method, $\delta_i = \delta_i\left(h\right)$ — dependence of the solution error on the step size h.

If the operation speed of calculating tool is set, and it is wanted to find a method that gives minimum error, the method should chosen with minimum value of

$$E_i = \delta_i\left(\frac{N_i}{S_3}\right), \tag{3}$$

where S_3 — preset operation speed.

In the general case, criteria (2) and (3) are not related, in other words optimum method for criterion (2) may be nonoptimal for criterion (3) and vice versa. However, under certain conditions that depend on the type of error functions $\delta_i\left(h\right)$, optimizing based on the discussed criteria leads to the same results.

It follows from the above that in order to determine the best method for numerical solution of dynamic equations one should have such characteristics of the methods as solution accuracy and amount of computation at the integration step. As for amount of computation, the major portion of the amount of computation falls on searching values of the vector function $f(y, U, p)$ of the system (1). Consequently, it is advisable to take hit count N for the procedure of calculation of the right hand sides of the system (1), that is required at each integration step while implementing the method, as a measure for the amount of computation at the integration step. Moreover, ratio N/h would serve as a measure for required computing tool operation speed.

Method of numerical experiment should be taken as an investigative technique for precision characteristics of various methods for solving differential equation systems that describe dynamics of an object. The reason is that analytical estimations of the accuracy of solving Cauchy problem obtained for various numerical methods are usually over evaluated, and often they are not calculated at all as they contain undetermined constants. The use of asymptotic estimates of local error, i.e. error at the integration step in order to compare methods as for their accuracy is improper just because of asymptotic nature, while transfer of the methods' properties from domain of extremely small integration steps to domain of final values can lead to gross errors in assessment of properties as for accuracy of different methods.

Advantage of the numerical experiment method consists in the ability to obtain objective characteristics of errors on the entire modelling range. Disadvantage of this method consists in partial nature of the results obtained, that are correct, generally speaking, only for the mode under consideration. In this regard, research of the accuracy properties of various numerical methods for solving differential equations of dynamics should be carried out in the most complex (extreme) dynamic modes. It can be predicted that accuracy of numerical methods in case of analysis of simpler regimes at least will not get worse.

In a computational experiment, information related to accuracy of various algorithms for solution of differential equations is obtained by comparing solution of particular problem with reference solution. Usually, for reference solution stands analytical solution that leads to comparison of the methods for simpler tasks (model problems). This approach is unacceptable for assessment of real accuracy, because transfer of results obtained for model problems that have analytical solutions may be impractical for real problems. Thus, for research according to the above considerations, it is necessary to obtain a reference solution for chosen dynamic mode. This problem can be solved using either the well-known Runge rule for assessment of accuracy of differential equation solutions or high accuracy order methods with assessment of local error at the integration step and its automatic choice.

Design of algorithms for approximate solution of differential equations. Currently, there are many well-known methods for numerical solution of ordinary differential equation systems, most of which can be divided into three groups: 1) one-step methods (such as Runge-Kutta methods) and static methods; 2) multistep finite difference methods (such as Milne, Adams methods etc.); 3) hybrid methods (such as Butcher method). All these methods are based on expansion of the sought-for vector function in power series and leaving in the latter a finite number of terms. Herewith the following condition is imposed: method of the r^{th} order should give accurate solution when the sought-for solution is an r^{th} degree polynomial. Presence of this condition, as well as presence of the well-known fact that any function which has sufficient number of derivatives and can be approximately replaced by an appropriate polynomial, determines the possibility of construction of these methods. Similar approach can be applied when using any closed system of functions (e. g. sine-cosine functions).

Another sufficiently general approach to the issue of construction of numerical methods for solution of differential equation systems consists in replacement of the initial system with another one, solution of which can be done quite simply with the use of analytical techniques. The best known methods used for implementation of this approach are so-called exponential methods, which essentially consist in presenting the right hand part of the resulting system as a sum of linear and nonlinear terms and using integral expression for linear systems. Variety of methods is obtained through various means of introduction of linear terms and through different integration techniques.

When choosing methods for numerical implementation of integral models, it is possible to use such characteristics as accuracy, operation speed and application area, which are shown in Table.

Table

Characteristics of methods for numerical implementation of integral models

Method name	Accuracy	Operation speed	Application area
Quadrature methods			
Degenerate kernel method	depends on quadrature formula	high	Real-time systems, solution of design optimization problems
Method based on combining closed Newton-Cotes quadrature formulas and Simpson quadrature formulas	$O(h^4)$	low	Analysis and synthesis of dynamic systems with insignificant error in input data

Continuation of Table

Method based on combining closed Newton-Cotes quadrature formulas	$O(h^8)$ — $O(h^{11})$	low	Analysis and synthesis of multichannel dynamic systems with insignificant error in input data
Iteration methods			
Simple iteration method	high	—	Linear dynamic systems design problems
Newton-Kantorovich method	high	—	Nonlinear dynamic systems design problems
Projection methods			
Adaptive collocation method	medium	medium	Analysis and synthesis of multichannel dynamic systems

Conclusions. Thus, the approaches discussed herein allow choosing the optimum method as for accuracy, amount of computation in phase of integration using analysis of the methods available. Implementation of this phase of research makes it possible to rank numerical implementation methods for different levels of acceptable error based on assessment of the value of the optimum criterion.

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Розглянуто підходи вибору оптимального методу щодо точності та обсягу обчислень на кроці інтегрування, що дає змогу на підставі оцінки значення критерію оптимальності зробити ранжування методів числової реалізації для різних рівнів допустимої похибки.

Ключові слова: *вибір числового методу, розв'язування рівнянь динаміки, алгоритми для вбудованих обчислювачів.*

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