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The authors of this paper study the modeling and improvement of communication systems using noise signals at modulation of parameters of non-Gaussian random processes. The algorithms of noise signals processing is synthesized when applying polynomial decision rules by the moment quality criterion of upper bounds of error probability. New mathematical models and methods of noise signals processing are developed. Simulation of the communication system was carried out in Simulink.

Key words: *Stochastic polynomials, moment quality criterion, discrimination noise signals, non-Gaussian noise.*

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A METHOD OF MODEL EXPERIMENTS FOR REGULARIZING THE PROBLEM OF RECOVERY OF THE INPUT SIGNAL OF THE LINEAR OBJECT BY THE INTEGRAL DYNAMIC MODEL

The method of determining the parameter of regularization for solving the problem of restoring the input signal of a linear stationary object under the conditions of the existence of measurement errors and the process of solving is considered. The proposed method of model experiments allows to separate errors of different types and restore the real input signal.

Key words: *integral model, Volterra equation of the I kind, external perturbation error, calculation method error, regularization.*

Introduction. The integral formulation of the problem of reconstructing the input signal of a stationary dynamic object is described by the Volterra equation of the I kind

$$\int_0^t K(t-s)\,y(s)\,ds = f(t), \tag{1}$$

where the functions $y(t)$ and $f(t)$ represent respectively the input and output signals, and the core (function $K(t)$) is the impulse response function of the object.

Problem of solving equation (1) belongs to a class of ill-posed problems, since the presence of errors in the right side and core usually causes numerical instability of the solution process, which makes it necessary to use regularization methods [1].

In many cases, in order to increase the stability of the solution process of the integral equation of type (1) A. N. Tikhonov's regularization method [2] is used, in which the problem is reduced to solving the integro-differential equation, one of the conditions for which is $K(0) \neq 0$. However, in real objects usually $K(0) = 0$, which limits the possibilities of the method.

To ensure stability of the computing process a possibility is considered of using the Lavrentiev's regularization method with a specific modernization of the regularization parameter's search process.

According to the Lavrentiev's method, instead of equation (1) the following equation is solved:

$$\alpha y(t) + \int_0^t K(t-s)\,y(s)\,ds = f(t). \tag{2}$$

Regularization process. The problem of determining the regularization parameter α is quite time- and effort-consuming. In the work [3] multiple ways of determining α are shown including the model experiments (examples) method for Fredholm integral equation of the I kind. Let us consider the possibility of applying the model experiments method to determine the regularization parameter α in the solution of Volterra integral equation of the I kind. Parameter α determination procedure consists of the following steps.

1. For the integral equation (1) to be solved a model equations is created

$$\int_0^t Q(t-s)\,y_Q(s)\,ds = f_Q(t), \tag{3}$$

in which $Q(t)$ coincides with the predetermined function $K(t)$ and solution $y_Q(t)$ is given (selected) in such a way, that the right side $f_Q(t)$ is as close as possible to $f(t)$, i.e.

$$f(t) \approx f_Q(t) = \int_0^t Q(t-s)\,y_Q(s)\,ds. \tag{4}$$

2. In practice, measurement mistakes are inevitable and instead of the equation with exactly known right side $\overline{f}(t)$ we have approximate right side, i.e.

$$f(t)=\overline{f}(t)+\Delta\overline{f}(t),$$

where $\Delta\overline{f}(t)$ — error. Having the law (eg, normal) of $\Delta\overline{f}(t)$ distribution, we can record

$$f(t)=\overline{f}(t)+\xi\overline{f}(t),$$

where ξ — normally distributed random number. Therefore, the model equation (3) we perturb with an error, at which values $\|\Delta f_Q\|/\|f_Q\|$ and $\|\Delta f_Q\|/\|f_Q\|$ are approximately equal, i.e. instead of equation (3) we have

$$\int_0^t Q(t-s)\overline{y}_Q(s)ds=f_Q^*(t), \qquad (5)$$

where $f_Q^*(t)=f_Q(t)+\xi f_Q(t)$.

Applicating Lavrentev's regularization method allows to record of equation (3) as

$$\alpha y_{Q\alpha}(t)+\int_0^t Q(t-s)y_{Q\alpha}(s)ds=f_Q^*(t). \qquad (6)$$

3. By repeated numerical solutions of (6), e.g. by using quadrature formulas, for a row of values of α $\alpha_{\mathrm{opt}Q}$ is determined, at which

$$\sum_{i=0}^{m}\left|y_{Q\alpha}(t_i)-y_Q(t_i)\right|^2=\min,\quad i=\overline{1,m}, \qquad (7)$$

where m — the number of sampling points.

4. The obtained value $\alpha_{\mathrm{opt}Q}$ is used to solve integral equation (1).

Numerical simulations show that using the modeling experiments method in the development of a numerical equation solving algorithm for signal restoration problem (1) allows to determine the effective values of parameter α, which regularizes the problem.

When numerically solving the considered problem, it is important to understand possible results' errors. Note that for a linear integral equation the error in solutions can be expressed using the errors' fundamental formulas. Indeed, the machine solution can be represented as depending on a number of quantities q_1, q_2, ..., q_n, characterizing the model's parameters, input actions, etc., deviations in which cause result's error. If there are deviations, real solution can be decomposed in the limited Taylor series and can be represented as

$$Y\left(t,q_1+\Delta q_1,\ldots,q_n+\Delta q_n\right)\approx Y\left(t,q_1,\ldots,q_n\right)+ \\ +u_1\left(t\right)q_1+u_2\left(t\right)q_2+\ldots+u_n\left(t\right)q_n, \tag{8}$$

where $u_1(t)$, $u_2(t)$, ..., $u_n(t)$ — influence (or sensitivity) coefficients.

Subtracting from (8) the exact solution of $Y(t, q_1, q_2, ..., q_n)$, we obtain

$$\Delta Y\left(t\right)+u_1\left(t\right)\Delta q_1+u_2\left(t\right)\Delta q_2+\ldots+u_n\left(t\right)\Delta q_n.$$

Sensitivity coefficients. To determine the sensitivity coefficients, it is possible to obtain the corresponding equations. We assume that the parameters q_1, q_2, ..., q_n are determined by the internal properties of the model, i.e. they are part of the core of the solved machine equation, which in this case has the form

$$\alpha Y\left(t\right)+\int_0^t K_M\left(t-s,q_1,\ldots,q_n\right)Y\left(s\right)ds=f\left(t\right). \tag{9}$$

($Y(t)$ — sought approximate solution)

Differentiating both sides of (9) by the parameter q_i (i = 1, ..., n), we obtain

$$\alpha\frac{\partial Y\left(t\right)}{\partial q_i}+\int_0^t\frac{\partial}{\partial q_i}K_M\left(t-s,q_1,\ldots,q_n\right)Y\left(s\right)ds=0$$

or

$$\alpha\frac{\partial Y\left(t\right)}{\partial q_i}+\int_0^t\left[\frac{\partial K_M\left(t-s,q_1,\ldots,q_n\right)}{\partial q_i}Y\left(s\right)+K_M\left(t-s,q_1,\ldots,q_n\right)\frac{\partial Y\left(s\right)}{\partial q_i}\right]ds=0.$$

Introducing the notation

$$\frac{\partial Y\left(s\right)}{\partial q_i}=u_i\left(t\right),\ \frac{\partial K_M\left(t-s,q_1,\ldots,q_n\right)}{\partial q_i}=K'_{Mq_i}\left(t-s,q_1,\ldots,q_n\right),$$

we obtain the sought equations

$$\alpha u_i\left(t\right)+\int_0^t K_M\left(t-s,q_1,\ldots,q_n\right)u_i\left(s\right)ds=-\int_0^t K'_{Mq_i}\left(t-s,q_1,\ldots,q_n\right)Y\left(s\right)ds. \tag{10}$$

As the function $Y(s)$, on the right side of (10) approximate solutions can be used. As seen from (10), to determine the sensitivity coefficients we can decide to use the basic solved equation, since the core of equation (10) coincides with its core.

As in the case of differential equations, for linear integral equations one can obtain an equation for the error. We assume that while solving (2), equation actually being solved looks like this

$$\tilde{y}\left(t\right)+\int_0^t\tilde{G}\left(t-s\right)y\left(s\right)ds=\tilde{\varphi}\left(t\right), \tag{11}$$

where $\tilde{G}(t-s)=\tilde{K}(t-s)/\alpha$, $\tilde{\varphi}(t)=f(t)/\alpha$. Function $\tilde{G}(t-s)$ accounts for initial modeling errors (methodical and instrumental errors), and is the sum

$$\tilde{G}(t-s)=G(t-s)+\Delta G(t-s).$$

Right side of equation (11) $\tilde{\varphi}(t)$ contains the external disturbance error and equals

$$\tilde{\varphi}(t)=\varphi(t)+\Delta\varphi(t);$$

$\tilde{y}(t)$ — approximate solution determined by the relation

$$\tilde{y}(t)=y(t)+\Delta y(t).$$

Where $\Delta y(t)$ — solution's total error. Then, by subtracting expression (2) from (11) we obtain

$$\Delta y(t)+\int_0^t\left\{\left[G(t-s)+\Delta G(t-s)\right]\left[y(s)+\Delta y(s)\right]-G(t-s)y(s)\right\}ds=\Delta\varphi(t).$$

Expanding brackets under the integral and considering errors $\Delta G(t-s)$ and $\Delta y(t)$ so small, that their product can be neglected, we obtain the sought equation

$$\Delta y(t)+\int_0^t G(t-s)\Delta y(s)ds=\Delta\phi(t)-\int_0^t \Delta G(t-s)y(s)ds$$

or

$$\alpha\Delta y(t)+\int_0^t K(t-s)\Delta y(s)ds=\Delta f(t)-\int_0^t \Delta K(t-s)y(s)ds.$$

It has to be considered, that using this equation to calculate the error $\Delta y(t)$ requires primary errors to be given in a certain form, and instead of the true solution $y(s)$ an approximate one should be used in the right part. Equation for the errors can also be used for the solution's quality evaluation, since it, in particular, shows that various components of the total error can be defined separately (leaving in the right side only $\Delta f(t)$, we can determine the result's inherited error, and leaving only the integral — numerical algorithm's error). In addition, the equation for the error allows us to make its assessment. Let us give an example of such an assessment.

If (t, s) belongs to the region D, $0\le t\le\delta, 0\le s\le t$ and you can set constraints

$$\max_{(t,s)\in D}\left|K(t-s)\right|\le K,\quad \max_{(t,s)\in D}\left|\tilde{K}(t-s)\right|\le\tilde{K},$$

$$\max_{(t,s)\in D}\left|\Delta K(t-s)\right|\le\delta,\quad \max_{t\in[0,\delta]}\left|\tilde{f}(t)\right|\le f,$$

$$\max_{t\in[0,\delta]}\left|\Delta f\left(t\right)\right| \le \eta,$$

then, using the results of [4], we obtain the estimate

$$\Delta y\left(t\right) \le \left[f\delta \frac{e^{\frac{1}{\alpha}\left(K-\tilde{K}\right)t}-1}{K-\tilde{K}} + \mu \right] e^{\frac{1}{\alpha}\tilde{K}t}.$$

Conclusion. Thus, the use of the Lavrentiev's regularization method in solving Volterra integral equations of the I kind provides required stability of the signal recovery process, and model experiments method allows determining the values of the regularization parameter. Expressions, obtained on basis of the accuracy analysis of solved equations, are the basis of deterministic and probabilistic error estimates of the sought solution.

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Розглядається спосіб визначення параметру регуляризації для розв'язання задачі відновлення вхідного сигналу лінійного стаціонарного об'єкту в умовах існування похибок вимірювання та процесу розв'язування. Запропонований метод модельних експериментів дозволяє відокремлювати похибки різних типів і відновлювати справжній вхідний сигнал.

Ключові слова: *інтегральна модель, рівняння Вольтерри I роду, похибка зовнішнього збурення, похибка методу обчислення, регуляризація.*

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