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IMPLEMENTATION OF INTEGRAL EXPLICIT MACROMODELS BY MEANS OF QUICK-ACTING ALGORITHMS

A class of mathematical models of dynamic objects in the integral macromodels form, built on the «input-output» principle is considered. The possibility of reducing errors and increasing speed of the modeling process using quadrature formulas based on integral macromodels in the Volterra operators form (Volterra-Hammerstein) is investigated. The constructive algorithms of numerical modeling procedures are proposed using the method of dividing kernels.

Key words: *macromodels, integral operators, quadrature formulas, simulation error, computational algorithm, speed.*

Introduction. Integral macromodels with a sufficient degree of adequacy for practical applications display processes in dynamic objects of varying complexity, assuming their representation by the «input-output» principle [1]. The statement of the modeling problem is set as follows:

- the dynamic characteristic is specified by $K(t,\tau)$ of a linear part, nonlinear dependence $F[\cdot]$ (for nonlinear objects) and the input signal $x(t)$;
- it is needed to identify the output signal $y(t)$, accordingly using integral macromodels of linear and nonlinear objects:

$$y(t)=\int_0^t K(t,\tau)x(\tau)d\tau\,, \tag{1}$$

$$y(t)=\int_0^t K(t,\tau)F\left[x(\tau)\right]d\tau\,. \tag{2}$$

In the relevant literature, integral macromodels of the form (1), (2) are known as the Volterra operators [2–11].

The basis of the integral operators' and the Volterra equations' numerical realization is the quadrature method, which consists [12, 13] in the application of quadrature formulas for the approximate calculation of a definite integral:

$$\int_a^b f(x)dx=\sum_{i=1}^n A_i f_i(x_i)+R[f]. \tag{3}$$

where x_i — fixed abscissas (nodes) of the section $[a,b]$, while $x_i = a$, $x_n = b$; A_i — numerical coefficients or weighting multipliers; $R[f]$ — residual term (the approximation error). As a rule, $A_i > 0$, $\sum_a^b A_i = b - a$.

There are a significant number of quadrature formulas of the form (3), which include the Newton-Cotes formulas (also rectangles, trapeziums, Simpson), Gauss, Chebyshev, etc. [12–14]. However, the use of the quadrature method in the case of arbitrary type kernels is related with the accumulation the number of operations on each discrete step, which in turn leads to the errors accumulation and the decrease in the modeling process speed.

Aim of the work. Research of the possibility of improving the quality of dynamic objects simulation by using the quadrature method in the case of using the dividing form kernels.

Dynamic Objects Simulation. The traditional approach to ensuring the accuracy of the quadrature method is based on selection of the discretization step and the form of the quadrature formula. For example, the required form of the quadrature formula can be obtained if we divide the integration interval into parts and apply our quadrature formula to each of them separately. Obviously, when dividing the initial section into parts, we should start from the condition that the integral of the resulting curve should be as close as possible to the integral from the function $f(x)$. After this, the values of the integral from the function $f(x)$ on the whole interval $[a,b]$ is defined as the sum of the found integral values from the function on separate parts of the total section $[a,b]$.

Error minimization (increasing precision) of the quadrature formula on the chosen functions class is achieved by choosing the quadrature-shape coefficients and by choosing the integration nodes. Depending on the usage, the nodes x_i can be selected in different ways. For trapeziums formulas, Simpson and Boole, it is recommended [12] to select equidistant nodes. For the Gauss-Legendre quadrature, the selected nodes should be the «zeros» of definite Legendre polynomials. The smaller the step of dividing a segment is selected $[a,b]$, the more precisely result will be obtained, but the number of computational operations increases, which requires additional resources. An important feature of the calculations in this case is *errors accumulation* with an increase of the number of steps, which is defined not so much by the step magnitude and the calculation precision on it, as by the «successful» or «unsuccessful» choice of the replacing

integral method by the final total. Such situation is pertained to the process of modeling objects in real time, when the integration interval (section $[a,b]$) could be large or previously unknown.

Next, we will use the frequently used representations of integral operators in the form:

$$f(x)=\int_a^b K(t,\tau)F\left[\tau,f(\tau)\right]d\tau\,, \tag{4}$$

called Volterra-Hammerstein [15, 16], and the approximation of the integrand expression written as follows:

$$f(x_i)=\int_a^{x_i} K(x_i,\tau)y(\tau)d\tau\,,\ x\in[a,b]. \tag{5}$$

It is worth noting that the property of increasing the volume of calculations with increasing the step relates to the case of the kernels with random form. The computation rate increase can be achieved using the degenerate dividing kernels method [17], which feature is the constant calculations amount in the step. Using this feature, we present the Volterra integral operator with a separating kernel:

$$\begin{aligned}\int_a^x K(x,\tau)y(\tau)d\tau &= \int_a^x \sum_{i=1}^m \alpha_i(x)\beta_i(\tau)y(\tau)d\tau = \\ &= \sum_{i=1}^m \alpha_i(x)\int_a^x \beta_i(\tau)y(\tau)d\tau.\end{aligned} \tag{6}$$

After approximating the integral (6) and usage of quadrature sums, we obtain:

$$\begin{aligned}&\int_a^{x_i} K(x,\tau)y(\tau)d\tau = \\ &= \sum_{i=1}^m \alpha_i(x)\int_a^{x_i}\beta_i(\tau)y(\tau)d\tau = \sum_{i=1}^m \alpha_i(x)\sum_{j=0}^n \beta_i(x_j)y(x_j) = \\ &= \sum_{i=1}^m \alpha_i(x)\left[\beta_i(x_n)y(x_n)+\sum_{j=0}^{n-1}\beta_i(x_j)y(x_j)\right] = \\ &= \sum_{i=1}^m \alpha_i(x)\beta_i(x_n)y(x_n)+\sum_{i=1}^m \alpha_i(x)\sum_{j=0}^n \beta_i(x_j)y(x_j).\end{aligned} \tag{7}$$

The value $\sum_{i=1}^m \alpha_i(x)\sum_{j=0}^n \beta_i(x_j)y(x_j)$ is known because it's calculated in the previous $(i-1)$-s step. Therefore, it remains to calculate the val-

ue of the sum $\sum_{i=1}^{m}\alpha_i(x)\beta_i(x_n)y(x_n)$, that requires an unchanged number of operations at each i -s step.

The significant part of integral operators and equalities encountered in practice have difference kernels presented in analytical form and having the property of separation.

For macromodels, for example, of the form (2), the quadrature method leads to the following calculated expressions (depending on the structural arrangement of the linear part in the general object structure or the absence of nonlinearity):

$$\tilde{y}(t_i)=\sum_{j=1}^{i}A_jK(t_i,\tau_j)F\left[x(\tau_j)\right], \tag{8}$$

$$\tilde{y}(t_i)=F\left[\sum_{j=1}^{i}A_jK(t_i,\tau_j)\right], \tag{9}$$

$$\tilde{y}(t_i)=\sum_{j=1}^{i}\left[A_j\,K(t_i,\tau_j)\right]. \tag{10}$$

For the case of a degenerate kernel:

$$\tilde{y}(t_i)=\sum_{i=1}^{m}\alpha_i(t_i)\sum_{j=1}^{i}A_j\beta_i(\tau_j). \tag{11}$$

If we introduce a nonlinear dynamic object with input and output signals, accordingly $\varphi(t)$, $\psi(t)$ while using a macromodel:

$$\psi(t)=\sum_{i=1}^{m}\alpha_i(t)\int_0^t\beta(\tau)F\left[\varphi(\tau)\right]d\tau, \tag{12}$$

then, according to the quadrature's method, it's possible to construct an algorithm for implementing the macromodel, shown in Fig.1. The algorithm performs calculations using formula:

$$\psi_i=h\sum_{i=l}^{m}\alpha_{li}\sum_{j=0}^{i}\left\{A_j\beta l_j\,F\left[\varphi_j\right]\right\}, \tag{13}$$

where $\alpha_{li}=\alpha_l(t_i)$; $\beta_{li}=\beta_l(t_j)$; h — discretization step; A_j — quadrature formula's coefficients; $\psi_i=\psi(t_i)$, $\varphi_j=\varphi(t_j)$.

Example. Suppose that on the interval $[0,1]$ with step $h=0,01$ there is a task to implement a nonlinear integral operator.

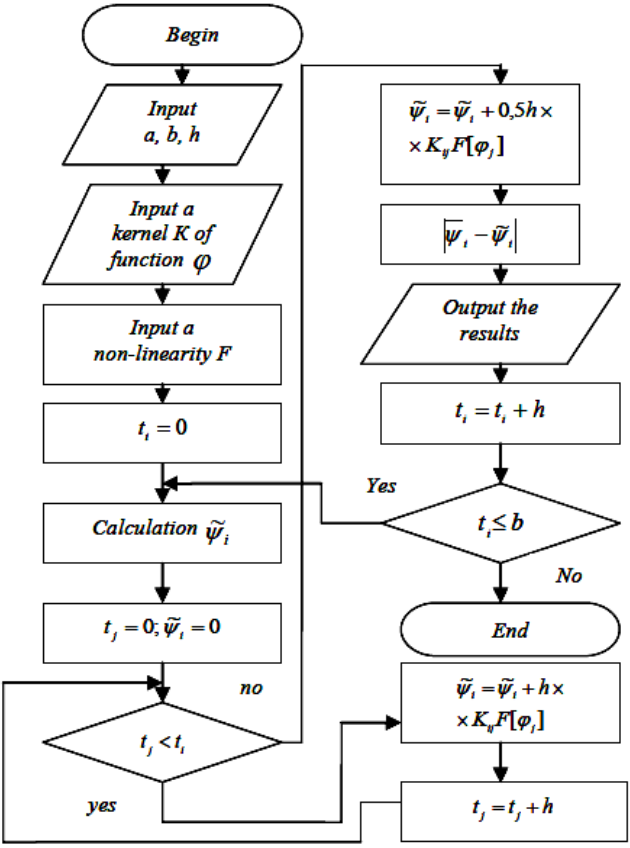

Fig. 1. *The algorithm for implementing a macromodel, constructed in accordance with the method of quadratures for a kernel of random form*

$$\psi(t) = \int_0^t e^{-a(t-\tau)} \sin^2(\omega\tau)\, d\tau, \qquad (14)$$

where $a = 0{,}1$; $\omega = 0{,}01$.

The exact value of the function sought is given by:

$$\psi(t) = \frac{1}{a^3 + 4\omega^2}\left[a\sin^2(\omega t) - \omega\sin(2\omega t)\cdot\left(1 - e^{-at}\right)\right]. \qquad (15)$$

The calculated ratio obtained based on the trapeziums formula for the numerical realization of the operator (12) with a non-divided kernel has the form:

$$\tilde{\psi}_1(t_i) = h\sum_{j=0}^{i} A_j l^{-a(t_i - t_j)} \cdot \sin^2(\omega t_j). \quad (16)$$

Using the kernel separability property, we obtain:

$$\tilde{\psi}_1(t_i) = h\sum_{l=0}^{m} l^{-at_i} \sum_{j=0}^{i} A_j l^{-at_j} \cdot \sin^2(\omega t_j). \quad (17)$$

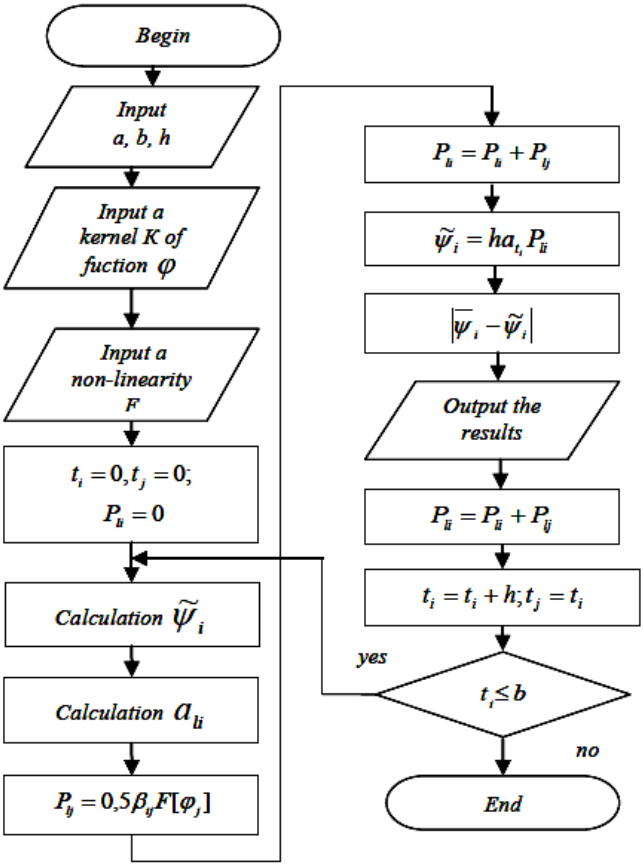

Fig. 2. *The algorithm that implements realization of the Volterra* Hammerstein integral operators with a divided kernel

The calculations results are given in Table. 1.

The second column shows the results of exact solution, the 3-rd and 4-th columns represent approximate calculation results $\tilde{\psi}(t_i)$ by algorithms, shown in Fig. 1 and Fig. 2, respectively.

Table 1

Exact and approximate solutions of the problems of the form (12)

t_1	$\tilde{\psi}(t_1)$	$\tilde{\psi}_1(t_1)$	$\tilde{\psi}_2(t_1)$	$\tilde{\psi}(t_1)-\tilde{\psi}_1(t_1)$	$\tilde{\psi}(t_1)-\tilde{\psi}_2(t_1)$	M_1	M_2
0,00	0	0	0	0	0	1	0
0,10	$0{,}332*10^{-5}$	$0{,}324*10^{-5}$	$0{,}434*10^{-5}$	$0{,}782*10^{-6}$	$0{,}167*10^{-6}$	56	6
0,20	$0{,}265*10^{-4}$	$0{,}263*10^{-4}$	$0{,}266*10^{-4}$	$0{,}161*10^{-5}$	$0{,}336*10^{-6}$	112	6
0,30	$0{,}893*10^{-4}$	$0{,}890*10^{-4}$	$0{,}894*10^{-4}$	$0{,}244*10^{-5}$	$0{,}507*10^{-6}$	168	6
0,40	$0{,}211*10^{-3}$	$0{,}210*10^{-3}$	$0{,}212*10^{-3}$	$0{,}326*10^{-5}$	$0{,}679*10^{-6}$	224	6
0,50	$0{,}411*10^{-3}$	$0{,}409*10^{-3}$	$0{,}413*10^{-3}$	$0{,}408*10^{-5}$	$0{,}857*10^{-6}$	280	6
0,60	$0{,}708*10^{-3}$	$0{,}707*10^{-3}$	$0{,}709*10^{-3}$	$0{,}490*10^{-5}$	$0{,}102*10^{-5}$	336	6
0,70	$0{,}112*10^{-2}$	$0{,}110*10^{-2}$	$0{,}113*10^{-2}$	$0{,}572*10^{-5}$	$0{,}120*10^{-5}$	393	6
0,80	$0{,}167*10^{-2}$	$0{,}160*10^{-2}$	$0{,}168*10^{-2}$	$0{,}653*10^{-5}$	$0{,}138*10^{-5}$	448	6
0,90	$0{,}237*10^{-2}$	$0{,}234*10^{-2}$	$0{,}238*10^{-2}$	$0{,}734*10^{-5}$	$0{,}155*10^{-5}$	504	6
1,00	$0{,}324*10^{-2}$	$0{,}321*10^{-2}$	$0{,}325*10^{-2}$	$0{,}817*10^{-5}$	$0{,}173*10^{-5}$	560	6

As can be seen from Table. 1, calculation by the algorithm that implements the expression (13) gives the minimum error. This follows from the fact that the number of arithmetic operations M_2 with increase of the number of the discretization node remains unchanged, while the number of arithmetic operations M_1 at numerical realization of the integral operator on the first algorithm grows.

The quantitative estimation of the solutions' accuracy is given in the Table 2 by calculated ratios of residual terms of the quadrature formula of the form (3), using representation of the Newton-Cotes (for the case when the initial point of the section coincides with the interpolation node).

Table 2

Residual members of the quadrature Newton-Cotes formula

n	$R[f]$	n	$R[f]$
2	$-\left[(1/12)h^3 f''\right]$	7	$-\left[(9/1400)h^9 f^{VIII}\right]$
3	$-\left[(1/90)h^5 f^{IV}\right]$	8	$-\left[(8183/518400)h^9 f^{VIII}\right]$
4	$-\left[(3/80)h^5 f^{IV}\right]$	9	$-\left[(2368/467775)h^{11} f^{X}\right]$
5	$-\left[(8/945)h^7 f^{VI}\right]$	10	$-\left[(4671/394240)h^{11} f^{X}\right]$
6	$-\left[(275/12096)h^7 f^{VIII}\right]$	11	$-\left[(673175/163459296)h^{13} f^{XI}\right]$

It should also be noted, that the algorithms shown in Fig. 1 and Fig. 2, have a close computational complexity.

Conclusion. A quick-acting algorithm for numerical realization of dynamic macromodels in the form of integral operators with divided kernel is proposed, which provides real-time calculations. The algorithm can serve as a basis for the creating of quick-acting specialized calculators for solving control and monitoring problems as well as for modeling a wide class of dynamic objects and processes. Through the given computational experiments, the algorithm confirms the constructiveness of the proposed approach.

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РЕАЛІЗАЦІЯ ІНТЕГРАЛЬНИХ МАКРОМОДЕЛЕЙ ЯВНОГО ВИДУ ЗАСОБАМИ ШВИДКОДІЮЧИХ АЛГОРИТМІВ

Розглянуто клас математичних моделей динамічних об'єктів у формі інтегральних макромоделей, що побудовані на принципі «вхід-вихід» Досліджується можливість зменшення помилок та підвищення швидкості процесу моделювання з використанням квадратурних формул на основі інтегральних макромоделей у формі операторів Вольтерра (Вольтерра-Хаммерштейн). Запропоновано конструктивні алгоритми процедур чисельного моделювання, використовуючи метод росчеплення ядер.

Ключові слова: *макромоделі, інтегральний оператор, квадратурні формули, похибка моделювання, обчислювальний алгоритм, швидкодія.*

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МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ КОЛИВНИХ ПРОЦЕСІВ У НЕОБМЕЖЕНОМУ КУСКОВО-ОДНОРІДНОМУ КЛИНОВИДНОМУ СУЦІЛЬНОМУ ЦИЛІНДРІ

Актуальність теорії крайових задач для диференціальних рівнянь з частинними похідними, яка інтенсивно розвивається, обумовлена як значимістю її результатів для розвитку багатьох розділів математики, так і численними застосуваннями її досягнень при математичному моделюванні різних процесів і явищ фізики, механіки, біології, медицини, економіки, техніки.

Добре відомо, що складність досліджуваних крайових задач суттєво залежить від коефіцієнтів рівнянь та геометрії області в якій розглядається задача. На цей час досить детально вивчено властивості розв'язків крайових задач для лінійних, квазілінійних та певних класів нелінійних рівнянь в однозв'язних областях.

Водночас багато важливих прикладних задач теплофізики, термомеханіки, теорії пружності, теорії електричних кіл, теорії коливань приводять до крайових задач для диференціальних рів-нянь з частинними похідними не тільки в однорідних об-