NUMERICAL COMPLEX ANALYSIS METHOD FOR PARAMETERS IDENTIFICATION OF ANISOTROPIC MEDIA USING APPLIED QUASIPOTENTIAL TOMOGRAPHIC DATA.
PART 2: ALGORITHM AND NUMERICAL EXPERIMENT

An algorithm, which lies in the sequential iterative applying of numerical quasiconformal mapping methods for constructing a series of dynamic meshes using different boundary conditions (that determined by experimental data) and solving the problem of parameter identification for each of these meshes is developed. It is based on the proposed approach to the solving of gradient problems of parameters identification of quasiideal fields with using applied quasipotential tomographic data in cases of anisotropic media and applying the ideas of the block iteration method. The reconstructed image of the distribution of conductivity tensor inside the investigated object, obtained as a result of numerical calculations performed on the basis of the developed algorithm with a sufficient accuracy corresponds to the etalon. The method is characterized by comparatively fast computer convergence (since, unlike many used methods, it does not require finding derivatives of the conductivity tensor distribution function at certain points and refining the boundary nodes at each iteration step). Its significant feature is the possibility of comparatively easy its paralleling and stopping the calculation procedure when some conditions for finishing the process are complete with simultaneous automatic determination the areas of the physical domain where have place large errors of the calculations, which makes it possible to use the machine time more economically. The algorithm for image reconstruction could be extended not only for the medium with a known sum of eigenvalues of the conductivity tensor, but also to cases of other rather wide dependencies between them. In particular, this approach provides an opportunity to represent it as some complex function as required by biomedical practice.

Key words: applied Quasipotential Tomography, Quasiconformal Mappings, Anisotropy, Identification, Nonlinear Problems.

Introduction. In the paper [1], the approach to the solving of gradient problems of parameters identification of quasiideal fields with using applied quasipotential tomographic data based on numerical complex analysis methods is transferred to cases of anisotropic media. In this, the
additional information about the nature of the conductivity distribution inside the domain (research object) is considered a priori known. However, in opposite to the traditional approaches to the statement and solving the problems of electrical impedance tomography, here set the local velocities distribution of a substance (liquid, current) in addition to the averaged potential at the contact sections of plate and body and at other sections (stream lines) here set the potential distribution (according to experimental data). Generation of initial data at the boundary of the investigated object is carried out in accordance with the polar model of current injection when eigenvalues sum of the conductivity tensor (CT) of the media is given. The corresponding problem is reduced to the iterative solving of a series of problems for Laplace type equations, where instead of «boundary numerical analogues of the Cauchy–Riemann type equations» appear the ratio of quasiorthogonality with using special types of optimization conditions. This work is devoted to the construction of an appropriate algorithm and conducting computer experiments.

**The algorithm.** Algorithm for solving the input problem lies in rotational parametrization of internal nodes of the mesh domains $G_z^{y(p)}$, CT and using an ideas of block iteration method [5, 6]. In particular: we set the number of injections $\bar{p}$, bound of domains $G_z^{y(p)}$ (by the functions $x = \tilde{x}(\tau), \ y = \tilde{y}(\tau)$), parameters $\tau_A^{(p)}, \ \tau_B^{(p)}, \ \tau_C^{(p)}, \ \tau_D^{(p)}$ and $\varepsilon_1, \varepsilon_2$ (of accuracy), $q$ ($q > 1$ is responsible for the number of iterations for correct of internal nodes having specific CT), quasipotentials $\varphi_s^{(p)}, \ \varphi^*(p)$ and discharges $Q^{(p)}$, parameters $m^{(p)}, \ n^{(p)}$ of $G_\alpha^{y(p)}$ partition (it is desirable to select this values so that $Q^{(p)} / (\varphi^*(p) - \varphi_s^{(p)}) / (m^{(p)} + 1) / (n^{(p)} + 1) \approx 1$ in order to improve the accuracy of the calculations) [3, 5], constants of functional (18) [1] $\mu$ and $\eta$, distribution of the eigenvalues sum $\lambda = \lambda(x, y)$ and parameters $\alpha_k$ ($1 \leq k \leq 4$) for inequalities-restrictions (19) [1]. Along with this we calculate the coordinates of the angular points $A_p = (\tilde{x}(\tau_A^{(p)}), \tilde{y}(\tau_A^{(p)})), B_p = (\tilde{x}(\tau_B^{(p)}), \tilde{y}(\tau_B^{(p)})), C_p = (\tilde{x}(\tau_C^{(p)}), \tilde{y}(\tau_C^{(p)})), D_p = (\tilde{x}(\tau_D^{(p)}), \tilde{y}(\tau_D^{(p)}))$ on $\partial G_z^{y(p)}, \ \Delta \varphi^{(p)} = (\varphi^*(p) - \varphi_s^{(p)}) / (m^{(p)} + 1), \ \Delta \psi^{(p)} = Q^{(p)} / (n^{(p)} + 1)$ and values of quasiconformal invariants $\gamma^{(p)} = \Delta \varphi^{(p)} / \Delta \psi^{(p)}$. 

12
Then we set the values of local velocities \( \Psi_{ij}^{(p)} \), \( \Psi_{j}^{(p)} \) (and therefore, stream functions \( \psi_{ij}^{(p)} \), \( \psi_{j}^{(p)} \)) and potentials \( \varphi_{i}^{(p)} \), \( \varphi_{i}^{(p)} \) having some arguments \( \tau = \tau_{ij}^{(p)} \), \( \tau_{j}^{(p)} \), \( \tau_{i}^{(p)} \), \( \varphi_{i}^{(p)} \) (results of physical measurements), respectively, after which we calculate (10) [1] using interpolation and finally we find the coordinates of \( x_{0,j}^{(p)} \), \( y_{0,j}^{(p)} \), \( x^{(p)}_{i,0} \), \( y^{(p)}_{i,0} \), \( x^{(p)}_{i,n^{p}+1} \), \( y^{(p)\;i,n^{p}+1} \), \( x^{(p)}_{m^{p}+1,j} \), \( y^{(p)}_{m^{p}+1,j} \) \( (1 \leq p \leq \tilde{p}, \; 0 \leq i \leq m^{p}+1, \; 0 \leq j \leq n^{p}+1) \) on \( \partial G^{(p)} \). Then we form the initial approximations of the nodes \( x_{i,j}^{(p,0)} \), \( y_{i,j}^{(p,0)} \) and list of parameters \( a^{(0)}_{k_{a}-r_{a},r_{a}}, \; b^{(0)}_{k_{b}-r_{b},r_{b}}, \; c^{(0)}_{k_{c}-r_{c},r_{c}} \), which define CT. After that we start the iterative process of reconstruction, which consists of the following steps: we apply the difference representation of Laplace type equations (14) [1] (with consider «injectivity») for search the coordinates of internal nodes when \( 1 \leq p \leq \tilde{p}, \; 1 \leq i \leq m^{p}, \; 1 \leq j \leq n^{p} \) \( q \) times; we solve the functional minimizing problem (18) [1] under conditions (19) [1] relative \( a^{(l)}_{k_{a}-r_{a},r_{a}}, \; b^{(l)}_{k_{b}-r_{b},r_{b}}, \; c^{(l)}_{k_{c}-r_{c},r_{c}} \) (here \( l = 0,1,... \) is the iterative step number, \( k_{a} = 1, s_{a} \), \( r_{a} = 0, k_{a} \), \( k_{b} = 1, s_{b} \), \( r_{b} = 0, k_{b} \), \( k_{c} = 0, s_{c} \), \( r_{c} = 0, k_{c} \)); check the conditions for the completion of the iterative process, among which may be [5]: stabilizing of near-boundary nodes, CT, the quasiconformal degree parameter, the values of discharges, etc. \( (1 \leq p \leq \tilde{p}, \; 1 \leq i \leq m^{p}, \; 1 \leq j \leq n^{p}) \). In the cases when one of these conditions is not satisfied the iterative process begins again, otherwise we build the corresponding reconstructed image and, if it’s necessary, the electrodynamical mesh, the complex quasipotential domains or calculate the velocity fields etc.

Note that the algorithm will be identical if instead of eigenvalues sum (4) [1], it is known the distribution either \( \lambda_{1} \) or \( \lambda_{2} \). In first case the term \( \tilde{\mu}(\lambda_{ij}^{\gamma(p)} - \sigma^{\gamma(p)}_{11,ij} - \sigma^{\gamma(p)}_{22,ij})^{2} \) of functional (18) [1] is replaced by

\[
\tilde{\mu} \left( \lambda_{1,ij}^{\gamma(p)} - 0.5 \left( \sigma^{\gamma(p)}_{11,ij} + \sigma^{\gamma(p)}_{22,ij} + \sqrt{\left( \sigma^{\gamma(p)}_{11,ij} - \sigma^{\gamma(p)}_{22,ij} \right)^{2} + 4 \sigma^{\gamma(p)2}_{12,ij}} \right) \right)^{2},
\]

in the second it is replaced by \( \tilde{\mu} \left( \lambda_{2,ij}^{\gamma(p)} - 0.5 \left( \sigma^{\gamma(p)}_{11,ij} + \sigma^{\gamma(p)}_{22,ij} - \sqrt{\left( \sigma^{\gamma(p)}_{11,ij} - \sigma^{\gamma(p)}_{22,ij} \right)^{2} + 4 \sigma^{\gamma(p)2}_{12,ij}} \right) \right)^{2} \). However, the solving of the nonlinear
programming problem thus created will require the use of the global optimization method.

In order to use the machine time more frugally, it is also possible to apply formulas (18) [1] and (19) [1] only for selected points. In particular (if it allows the chosen optimization algorithm) the fulfillment of conditions (19) [1] should not be required in all nodes of the \( \tilde{p} \) meshes, but only in the coordinates of the extreme values of the functions (4) [1] instead. It makes sense to set a series of control points inside the investigated domain in other cases. Such in some cases may be nodes of meshes from arbitrary injection.

Also note, that instead of the procedure for determining the coordinates of the boundary nodes using formula (16) [1] (by interpolating the results of physical measurements), we can immediately select them so that the local differences in the values of the function of flow or potential between them at the corresponding neighboring points to be constant within the injection.

**Numerical calculations.** We represent the results of numerical calculations for the following input data: 

\[
\begin{align*}
  s_a & = 2, & s_b & = s_c & = 3, & \tilde{p} & = 20, & \tilde{x}(\tau) = 150 \cos \tau, & \tilde{y}(\tau) = 100 \sin \tau, & a_{k_a - r_a, r_a} &= b_{k_b - r_b, r_b} = c_{k_c - r_c, r_c} = 0 & (k_a = 1, s_a), \\
  r_a & = 0, k_a, & r_b & = 1, s_b, & r_b & = 0, k_b, & r_c & = 0, s_c, & r_c & = 0, k_c, & a_{0,0} = b_{0,0} = 1, \\
  \mu & = 0.1, & \eta & = 0.01, & \alpha_1 & = \alpha_2 & = 0.01, & \alpha_3 & = \alpha_4 & = 4, & q & = 200, \\
  \varepsilon_1 & = \varepsilon_2 = 10^{-2}, & m^{(p)} & = 100, & \varphi^{(p)} & = 0, & \varphi^{*(p)} & = 1, & \tau_A^{(p)} & = 9\pi/8 + (p - 1)\pi/\tilde{p}, & \tau_B^{(p)} & = \tau_A^{(p)} - \pi/4, & \tau_C^{(p)} & = \tau_A^{(p)} - \pi, & \tau_D^{(p)} & = \tau_C^{(p)} - \pi/4, \\
  Q^{(p)}, & \Psi_s^{(p)}, & \Psi_j^{(p)}, & \varphi_i^{(p)}, & \varphi_i^{*(p)} & = 0 & (1 \leq p \leq \tilde{p}), & \lambda(x,y). \end{align*}
\]

Visual representation of the received CT distribution is carried out using a specially developed procedure similar to [4]. According to this, the investigated domain is divided into square sections by lines parallel to the axes of coordinates. The CT is characterized in the center of each of them as an ellipse (its directions of axes and radiuses are corresponds to the directions of eigenvectors and proportional to the eigenvalues, respectively) by the formula

\[
\kappa_1 x^2 + \kappa_2 y^2 + 2\kappa_{12} xy = 1, \quad \text{where} \quad \kappa_{11} = \sin^2 \theta / \lambda_2^2 + \cos^2 \theta / \lambda_1^2, \quad \kappa_{22} = \cos^2 \theta / \lambda_2^2 + \sin^2 \theta / \lambda_1^2, \quad \kappa_{12} = (\lambda_1^{-2} - \lambda_2^{-2}) \sin \theta \cos \theta, \quad \text{the angle} \ \theta \ \text{of rotation of the ellipse must satisfy the conditions} \ 2\sigma_{12} = (\lambda_1 - \lambda_2)\sin 2\theta, \quad \sigma_{11} = (\lambda_1 - \lambda_2)\cos^2 \theta + \lambda_2. \]

Figura. b presents the reconstructed image of the CT distribution in comparison to the given theoretically (Figura. a).
Conclusions. The algorithm for solving the problem of image reconstruction of the CT of anisotropic media given in [1] is developed. It is characterized by comparatively fast computer convergence (since, unlike many used methods, it does not require finding derivatives of the CT distribution function at certain points and refining the boundary nodes at each iteration step).

The significant feature of developed algorithm is the possibility of detection of so-named «stagnant zones» and «zones of large gradients», which appears near the especial points of non-smooth boundary lines and critical points inside the respective domains. We also note that the considerably new in algo-
A R I T H M is considering the conditions of «anisotropic quasirecthogonalit» along the boundary equipotentials and current lines (instead of orthogonality in cases of isotropic media), which requires additional substantially new constructions. Also, the anisotropy tensor affects the decrease in accuracy by orders of magnitude and stability, which in particular requires the creation of new structures-procedures for Tikhonov-type regularization.

We plan to extend the proposed algorithm to the following cases: when have place other rather wide dependencies between eigenvalues of the CT, spatial resolution, applying the quasipotential of the initial stream to several sections.

References:
задачі параметричної ідентифікації для кожної з цих сіток. Реконструйоване зображення розподілу тензора провідності у внутрішності досліджуваного об’єкта, отримане в результаті числових розрахунків, проведених на основі розробленого алгоритму, з достатньою точністю відповідає еталону. Метод характеризується порівняно швидкою комп’ютерною обчислювальністю (оскільки, на відміну від багатьох використовуваних методів, не потребує знаходження похідних функції розподілу тензора провідності у визначених точках та уточнення гранчичних вузлів на кожному ітераційному кроці). Суттєвою його особливістю є можливість порівняно легкого його розпаралелення та зменшення процесу обчислень за умови використання лише деяких із умов закінчення процесу з автоматичним визначенням тих ділянок фізичної області, де мають місце великі похибки обчислень, що дає змогу економізувати машинний час. Розроблений алгоритм реконструкції зображення може бути поширенений на інші задачі, але й на випадки досить широких інших залежностей між ними. Зокрема, алгоритм забезпечує можливість представлення його дуже комплексно значною функцією, яка вимагає біомедична практика.

Ключові слова: томографія прикладених квазіпотенціалів, квазіконформні відображення, анізотропія, ідентифікація, нелінійні задачі.

УДК 519.6:004.02

Л. П. Вакал*, канд. техн. наук,
Є. С. Вакал**, канд. фіз.-мат. наук

*Інститут кібернетики імені В. М. Глушкова НАН України, м. Київ,
**Київський національний університет імені Тараса Шевченка, м. Київ

Найкраще рівномірне наближення сплайнами з використанням диференціальної еволюції

Розглянуто задачу найкращого рівномірного наближення функцій поліноміальними сплайнами з фіксованими вузлами. Для її розв’язання запропоновано підхід на основі еволюційних алгоритмів — потужного класу стохастичних пошукових методів оптимізації. Для знаходження сплайна найкращого рівномірного наближення адаптовано алгоритм диференціальної еволюції. Це один із кращих еволюційних алгоритмів, який стабільно знаходить глобальний оптимум цільової функції (критерій оптимізації) за мінімальний час. Еволюційний процес в алгоритмі починається з генерації популяції випадкових векторів, координати яких представляють сьому можливі значення коефіцієнтів сплайна. Далі вектори постійно модифікуються за допомогою операцій мутації, схрещування та селекції з метою змен-