INCREMENTAL DIGITAL QUASI-IDEAL INTEGRATOR APPLICATION FOR ADVANCE FLUX ESTIMATION OF CONTROLLED INDUCTION MACHINE

The performance of the speed controlled induction machine principally depends on the accuracy of the estimated flux. The proposed method compensates the error produced by the inherent problem in the “pure” integrator and measurement error. This paper describes the problem associated with a quasi-ideal digital integrator in particularly a modern DDA-type (Digital Differential Analyzer) – an incremental digital integrator (IDI). The paper essentially discusses the development of the approach to the total error correction of DDA-type IDI. It is an element for processing incremental digital input-output signals using DDA principles. The basic types of errors of the incremental digital integrator are presented and then the reasons for their appearance are examined. The differential equation \(dY=aYdx\) as an example the quantitative relation of errors is investigated. The IDI error from the analytical solution is not exceeding one increment (quant) of sub-integral function \(Y\) even during a very long interval of integration variable \(x\). This means that the IDI becomes a practically ideal integrator. The suggested methods of correcting IDI errors can be applied in simulation, modeling, especially for dynamic systems control, etc. This method is easily applied in a DSP based induction machine control to estimate the flux.

**Key words:** induction machine, digital integrator, correcting errors.

I. Stator flux estimation problem of controlled induction machine

Flux estimation is the important part in induction machine regime control. The stator flux can be estimated from the measured terminal voltages and currents [8]. Once the stator flux is available it is easy to calculate the rotor flux.

The simplified three-axis model is transformed to two-axis model and then all the analysis is done in the two axes [9]. The D-Q model is shown in Fig. 1. The stator fluxes are calculated from Fig. 1 as:

\[
\lambda_{ds} = \int \left( V_{ds} - R_s i_{ds} \right) dt \\
\lambda_{qs} = \int \left( V_{qs} - R_s i_{qs} \right) dt
\]
**Fig. 1.** D-Q model of an induction machine (a) d-axis (b) q-axis

The integration given in this system of equations is usually implemented by analog or digital microprocessor technique [14, 15, 19, 20]. But in this paper is implemented by faster numerical incremental digital integrator (IDI) using the proposed quasi-ideal integrator approach in the next section. The d-axis flux and q-axis fluxes estimations implementation using the proposed scheme are shown in **Fig. 2.**

**Fig. 2.** The stator flux estimation using the proposed ideal IDI: 
(a) d-axis; b) q-axis flux estimation.

The magnitude of the stator flux is calculated as:

\[
\bar{\lambda}_s = \sqrt{\lambda_{ds}^2 + \lambda_{qs}^2}.
\]  

(2)

The stator flux is estimated using the proposed very well method. The rotor flux components can be calculated from the stator flux as presented below:

\[
\lambda_{dr} = \frac{L_r}{L_m} (\lambda_{ds} - L_s \sigma_i_{ds}).
\]
\[ \lambda_{qr} = \frac{L_r}{L_m} (\lambda_{qs} - L_s \sigma_i_{qs}), \]  

where \[ \sigma = 1 - \frac{L_m^2}{L_s L_r}. \]  

And the magnitude of the rotor stator flux will be calculated as:

\[ \bar{\lambda}_s = \sqrt{\lambda_{ds}^2 + \lambda_{qs}^2}. \]

All the above mentioned calculations are made every period of the applied alternating voltage V components V\(_d\) and V\(_q\) with recollection (correction) the preliminary initial condition in the IDI registers. It provides ideal integration of both IDI’s during every next period and real value and position of flux \(\lambda_s\) and use them in the control loop of the induction motor. This method is easily applied in a DSP (Digital Signal Processing) based induction machine control to estimate the flux. The closer to ideal integration the more correct the flux \(\lambda_s\) can be determine and the more effective will be control of the induction machine. The execution time required to implement the proposed system is small that there will not be any software computation burden.

**II. Incremental digital integrator**

Just to remained here, IDI is an incremental digital integrator comprising at least one arithmetic device for executing add and subtract (A/S) operations, and at least a one-two operand register, connected to the A/S device (Fig. 3). This technique utilizes so-called increment signals and following the DDA principles it processes the increments, which usually consist of two separate signals for value and sign.

![Fig. 3. Incremental Digital Integrator](image)

The output/input increment signals of IDI are often ternary ones, which in practice consist of two separate but associated, binary signals (a value and its sign). Together these describe the three states (+1, 0 –1). IDI processes the digital input-output increment signals, whose frequency is proportional to an analog quantity comprising a sign signal. The associated
sign signal indicates the polarity. When such a pulse frequency is applied to an up/down counter, whose counting-direction is determined by the sign signal, the value pulses are algebraically summed in accordance with the sign. Thus, integration is achieved, and this is presented in the counter as a binary number. Similarly, other known analog arithmetic and different nonlinear operations are processed by IDI. So, relatively simple incremental digital implementations of the control functions can be obtained. At the same time the computational errors are usually held within pre-determined limits.


The main disadvantage of IDI is the poorest integration method (simple Euler’s) and, so, the total error level is high. So, the correction of the IDI total error is an imperative problem to solve in this paper for control, analyzing and modeling structure based on IDI.

The error corrections of integration methods are widely used in digital modeling – Babuska, I. (1984), Giles, M. B. (2002), and in some (not IDI) type of digital control systems: for examples, Holtz J. (2002) and Seyoum M. (2003) use the correction of the simple Euler’s integration to improve flux estimation of induction machines in the microprocessor controlled speed system. But these methods are not directly applicable for IDI systems due to their hardware differences, the incremental nature of the data processing and the individualism for every particular system.

A few articles reported the methods for the correction of the IDI total error. For example, one of the earliest P. G. Ali-Zade (1968) and P.G. Ali-Zade et al. (1975) used an approach for the correction of the IDI structures total error by inserting initial value to the R-register. But in these papers the initial value of R-register was determined experimentally for particular initial value in Y-register by the trial-error method and only the total error correction of a single IDI (not the IDI-system) was discussed. This paper
develops the approach of the total error correction proposed by resulting with the simple linear expression for calculating an initial value in R-register. The applicability of the approach of the total error correction for IDI-system is also considered and the analytical formulae that bound initial values of Y- and R-registers are established. It will be seen that the truncation errors can be minimized and limited to almost zero-averaged error less than the least-significant bit of a Y-register (one quantum) to prevent drift effects due to side solutions and delays. Unlike truncation errors, round-off errors cannot be completely eliminated and can cause localized irreversibility of computations (hysteresis effects). In spite of this, in the control systems harmful drifts or accumulated errors are not produced because of the closed-cycle negative feedback nature, but can decrease their dynamic stability.

III. Correction of a single IDI

In considering the IDI integration process the differential equation (DE) \( \frac{dS}{dx} = Y \) is taken up as the IDI main equation, which simplifies modeling process and block-diagram design. However, the real mathematical model of the IDI with ternary increments is the following system of nonlinear difference equations: on the i-th step (iteration) increment of sub-integral function:

\[
\Delta Y_i = \sum_{j=1}^{l} \Delta S_{j,i-1};
\]

new value of sub-integral function:

\[
Y_i = Y_{i-1} + \Delta Y_i;
\]

new integer part of integral increment:

\[
\Delta S_i = \left[ R_{i-1} + Y_{i-1} \Delta x_i \right];
\]

new fractional part of integral increment:

\[
R_i = \left\{ R_{i-1} + Y_{i-1} \Delta x_i \right\},
\]

where \( l \) is a number of parallel acting IDIs, which are connected to Y input of the particular IDI; the presented “[ ]” are the brackets pick up the integer part of integral increment; “{ }” are the brackets pick up the fractional part of integral increment.

The mathematical analysis and solution of this nonlinear difference equations system (6-9) is very difficult. So we begin with the first-stage development of the methods for IDI correction with a detailed analysis of the total error of the solution for a certain IDI system. Next, we find such characteristics and then parameters for corrections, which are suitable for a wide range of coefficients of the system during a very long interval of integration variable \( x \).

Errors appearing at every step (iteration) of IDI integration can be divided according to the characteristics under discussion [2-3, 9]:
1. An integration method error (simple Euler’s method);
2. A truncation error (only the integer part of the increment is considered as IDI output on every $\Delta x$ step);
3. An error due to the different delays in the researched IDI system as a result of incidental-subsidiary solutions of higher-level difference equations (than the real system DE model);
4. A round-off error due to limitations of the binary registers in use.

Although truncation error is usually greatly predominates among them and this paper will consider the total error analysis of the IDI systems to find correction methods to compensate for the error.

A simple but effective method of compensating for the total error is to insert the initial conditions not only into the Y-register of the particular IDI-scheme of control system, but also into the IDI’s R-registers. The most commonly used and unsophisticated IDI-scheme consists of a one-two IDI, which solves the differential equation $dY = aYdx$ ($Y=Y_0\exp(ax)$) or, in other words, a one-two IDI with feedback. For the case of $a = 1$ only one IDI is needed as shown in Fig. 2.

Before determining the initial correction value ($R_0$) of the R-register to correct the total error, it is essential to state that:

1. Every unit inserted into any $k$-th bit of the Y-register produces an autonomous close to an exponential solution that is proportional to its weight: $2^{k-n}\Delta Yexp(x)$, where $n$- length of Y-register is in use. This statement is correct only for $Y_0\neq 0$ (usually $Y_0\neq 0$). The R-register is the extension of the Y-register (Fig. 2b). Thus, $k$ varies from 1 to $2n$. In this case the weights of the coefficients are equal to $2^{k-2n}$ for each bit of R-register and they are less than 1 ($2^{k-2n}<1$).

2. For the purpose of the detection error function (for $R_0=0$ and $a=1$), the IDI solution was compared with the ideal one $Y = Y_0\exp(x)$. The com-
parison (for different \( n \)) showed that the error changes are close to the exponential function: \( \Delta = Y_0 \exp(x) - Y(x) = r \Delta Y \exp(x) \), where \( r < 1 \).

3. Any changes of initial value (\( R_0 \)) in the R-register influence the total error of the IDI solution. Thus, the desired binary initial value of \( R_0 \approx r \), which is supposed to be inserted into the R-register, can significantly reduce the total error up to one increment of sub-integral function or the least-significant bit of a Y-register.

*Fig. 5* presents the total error of the IDI solution with a correction (curve 1 and \( R_0 = 0.5784 \)) and without a correction (curve 2 and \( R_0 = 0 \)) for \( Y_0=0.0313 \) and \( n=12 \). The vertical axis, representing the total error, was estimated in normalized form according to the following expression:

\[
\Delta = Y_0 2^n \left( \exp\left(\frac{x}{2^n}\right) - Y \right).
\]

(10)

*Fig. 5. IDI total error of the solution \( dY = Ydx \) with and without correction.*

As can be seen from *Fig. 5* (curve 1) for some \( R_0 \)-optimal the total error does not exceed one increment of sub-integral function or the least-significant bit of a Y-register.

To estimate the influence of \( R_0 \) for correcting the total error, the multi-variant experiments were conducted with different binary lengths for the Y- and R-registers \( (n=12, 13, 14, 15, \text{ and } 16 \text{ bits}) \). *Fig. 6* gives the results of these experiments: the curves agree closely with each other and the optimal values of \( R_0 \) are almost the same (between 0.577942 – 0.579711) as shown in *Table 1*. In *Fig. 6* the vertical axis is a root mean square error \( (J) \), calculated according to the expression:

\[
J = Y_0 2^n \sqrt{\frac{\sum_{i=1}^{m} (\exp(x_i) - Y_i)^2}{m}},
\]

(11)

where \( m \) is the number of integration steps needed to reach the \( Y=1 \).

The V-type curve in *Fig. 6* shows that there is a shaded region for \( R_0 \), where the total error does not exceed one increment of sub-integral function or the least-significant bit of a Y-register during a very long interval of integration.
Table 1

<table>
<thead>
<tr>
<th>bits</th>
<th>$R_0$</th>
<th>$J$</th>
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<tbody>
<tr>
<td>12</td>
<td>0.578369</td>
<td>0.3038426</td>
</tr>
<tr>
<td>13</td>
<td>0.579711</td>
<td>0.3033184</td>
</tr>
<tr>
<td>14</td>
<td>0.577942</td>
<td>0.3033623</td>
</tr>
<tr>
<td>15</td>
<td>0.578156</td>
<td>0.3028602</td>
</tr>
<tr>
<td>16</td>
<td>0.577285</td>
<td>0.3033923</td>
</tr>
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</table>

The $R_0$-optimal value corresponds to the minimum of RMSE and consequently to the minimum total error. However, as was found during multi-variant analyses, the initial value $R_0$ depends on the initial condition $Y_0$. The functional dependence $R_0$ versus $Y_0$ is shown in Fig. 7 with different binary lengths for Y- and R-registers $n = 12, 13, 14, 15$ and 16 bits.

![Fig. 6. Root mean square error (RMSE) versus $R_0$ for $Y_0 = 0.0313$](image)

![Fig. 7. The functional dependence $R_0$ versus $Y_0$](image)

There are three lines in Fig. 7: the upper one – majorant function, the lower one – minorant function and the optimal function is between them. For any magnitude of $Y_0$ the value $R_0$ between the upper and lower boundaries provides the total error that does not exceed the least-significant bit of a Y-register during a very long interval of integration.
The function between the majorant and minorant functions is the functional dependence of the $R_0$-optimal values (minimum RMSE) versus $Y_0$. The above-mentioned lines can be approximated by the linear functions (7):

$$R_0 = \alpha + \beta Y_0$$

Table 2

<table>
<thead>
<tr>
<th>$R_0$ (lower)</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.517 \pm 0.001$</td>
<td>$1.630 \pm 0.007$</td>
<td></td>
</tr>
<tr>
<td>$R_0$ (upper)</td>
<td>$0.517 \pm 0.001$</td>
<td>$2.639 \pm 0.007$</td>
</tr>
<tr>
<td>$R_0$ (optimal)</td>
<td>$0.517 \pm 0.001$</td>
<td>$1.889 \pm 0.008$</td>
</tr>
</tbody>
</table>

The coefficients of the linear functions in Table 2 were calculated with a confidence interval of 95%. The recommended correction method is reasonable only for a very long interval of integration. Further, because of the growing character of IDI solution only the narrow interval of the small initial values of $Y_0$ was investigated (as shown in Fig. 7).

The region between the upper and lower boundaries of $R_0$ is expanded (shaded in Fig. 7) as the initial value of the sub-integral function gets bigger. The difference between the majorant and minorant functions is approximately equal to the given initial magnitude of the Y-register ($Y_0$).

Thus, for any initial values of $Y_0$ and $n$ it is possible to find such an initial value $R_0$ that provides the integration process with the total error which does not exceed the least-significant bit of a Y-register during a very long interval of integration. In short, IDI becomes a nearly ideal integrator.

IV. Correction of IDI-system

To solve the differential equation $dY = a Y dx$, where $0 < a < \pm 1$, two IDIs are needed. So, one more IDI should be added to scale x with the coefficient $a$ (initial value of Y-register), named later $a$-IDI. One of the schemes for solving the above differential equation is shown in Fig. 8.

![Block-diagram for solving the differential equation $dY = a Y dx$](image-url)

**Fig. 8.** Block-diagram for solving the differential equation $dY = a Y dx$, where $0 < a < \pm 1$.

According to the block-diagram the $a$-IDI changes the frequency of the increment signal for the second IDI (dx input) and hence provides the scaling of the solution of the differential equation with the coefficient $a$. Implementing $a$-IDI significantly influences the parameters of the correc-
tion, which depend not only on the initial value of Y-IDI (Y-register), but also on the initial value of \( a \)-IDI (Y-register).

For the correction of the total error of the solution the R- registers are now available in both IDI. However, several preliminary experiments showed that the different initial values in the R-register of \( a \)-IDI did not essentially reduce the total error. As a consequence, setting the initial value in the R-register of Y-IDI was considered only for total error correction.

![Fig. 9. Initial value in R-register of Y-IDI versus initial value of \( a \)-IDI for \( Y_0 = 0.0313 \)](image)

To evaluate the effect of the initial value in the R-register of Y-IDI on the total error the following experiment was conducted:

1. The various initial values in the Y-register of Y-IDI were fixed in the same range 0.0156-0.0625, as used above.
2. For every fixed value in the Y-register of Y-IDI the initial value of \( a \)-IDI (coefficient \( a \)) was changed to the range 0.1-0.9.
3. The initial value in the R-register of Y-IDI for every combination of the variables in items 1 and 2 was evaluated to provide the total error of the solution that does not exceed the least-significant bit of Y-register of Y-IDI.

Fig. 9 shows an initial value in the R-register of Y-IDI versus initial value of \( a \)-IDI for one particular \( Y_0 = 0.0313 \) (Y-register of Y-IDI). The \( R_{\text{lower}} \) and \( R_{\text{upper}} \) represent the lower and upper boundaries of \( R_0 \) (the R-register Y-IDI), and provides the total error of solution that does not exceed the least-significant bit of Y-register (Y-IDI). There are some “resonant” points with a value \( a \) (0.125, 0.25, 0.5), where the lower boundary of R-register of Y-IDI is considerably reduced and reaches the value 0.5708, which is common for all “resonant” points of a particular \( Y_0 \) (Y-register of Y-IDI). It means that for the “resonant” points the region of \( R_0 \) is even wider for the total error correction. The range between the lower and upper boundaries of the R-register (Y-IDI) is enhanced, while the coefficient \( a \) is
increased. The middle curve (shown in Fig. 9 as $R_{\text{middle}}$) between the lower and upper boundaries can also be approximated by a linear equation.

As mentioned above, changing the initial value $Y_0$ of Y-IDI reflects a range of initial values in the R-register of Y-IDI and provides the correction of the total error in a limit of the least-significant bit of the Y-register of Y-IDI. Fig. 10 represents the family of curves for various initial values of Y-IDI. These curves show the same dependence of the initial value in the R-register of Y-IDI versus the coefficient $a$ as $R_{\text{middle}}$ in Fig. 7.

![Fig. 10. An initial value in the R-register of Y-IDI versus initial value of a-IDI for various initial values of Y-IDI](image)

A close examination of the curves shows that all of them can be approximated by a linear equation and the slope of lines is reduced; at the same time the initial value $Y_0$ of Y-IDI is increased. Thus, two parameters (the initial value $Y_0$ of Y-IDI and the coefficient $a$) determine the initial value $R_0$ of Y-IDI for a static correction of the total error. The following expression was established using a linear regression model, where the coefficients $\alpha$, $\beta$, $\gamma$ are shown in Table 3:

$$R_0 = \alpha + \beta \cdot Y_0 + \gamma \cdot Y_0 \cdot a.$$  \hspace{1cm} (8)

For $a = 1$ the expression (8) is converted to $R_0 = 0.518 + 2.595 \cdot Y_0$, which represents the line in the range between the lower and upper boundaries of $R_0$ established for the single IDI (see Fig. 7 and the expression (7)).

### Table 3

<table>
<thead>
<tr>
<th>Method</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequential</td>
<td>0.518 ± 0.002</td>
<td>1.576 ± 0.011</td>
<td>1.019 ± 0.009</td>
</tr>
<tr>
<td>Parallel</td>
<td>0.518 ± 0.002</td>
<td>1.565 ± 0.011</td>
<td>2.021 ± 0.009</td>
</tr>
</tbody>
</table>

The differential equation $dY = aY \, dx$, where $0 < a < \pm 1$, by using two IDIs in accordance with Fig. 10 can be solved as follows:
1) in a sequential manner (processing \textit{a}-IDI, then processing Y-IDI or processing Y-IDI, then processing \textit{a}-IDI);

2) in a parallel manner (processing both IDI in parallel).

The sequential methods need more time for processing both IDI, but reduce the memory needed for intermediate results. By contrast, a parallel approach reduces processing time, but needs more memory for intermediate results. For a parallel method the same expression (8) can be used with slightly changed coefficients (presented in Table 3).

Thus, the expression (8) provides the tool to set up the static correction by inserting an initial value \( R_0 \) into the R-register of Y-IDI. It is important to note that only the R-register, which is connected to the Y-register of Y-IDI can be used for a static correction of the total error. As closely analyzed the different initial values in the R-register of \textit{a}-IDI did not essentially reduce the total error because the R-register of \textit{a}-IDI is not the tail of the Y-register of Y-IDI (see Fig. 4b).

All the above mentioned calculations are made every period of the applied alternating voltage \( V \) components \( V_d \) and \( V_q \) with recollection corrected the preliminary \( R_0 \) in the IDI R-registers. It provides ideal integration of both IDI’s during every next period and real value and position of flux \( \lambda_s \) and use them in the control loop of the induction motor. This method can be easily applied in a DSP based induction machine control to estimate the flux. The execution time required to implement the proposed system is small that there will not be any software computation burden.

V. Conclusion

This paper describes the problem associated with a pure integration applied to DSP of the controlled IM stator flux estimation. Integration error comprises, due to different reasons, of a drift produced by the digital integrator, somehow related to initial condition, too.

IDI-technology is widely used in control of dynamic systems, image signal processing, and parallel large scale integration. Modern IDI is one of the fastest, most reliable, simple and tolerable technology. But, the main disadvantage of IDI is the poorest integration method and, as follows, the higher total error level. So, a correction of the IDI total error was a main concern for IDI-systems. The method of static correction applied in the paper is based on inserting additional initial values (\( R_0 \)) into the fractional part of incremental digital summer (the R-register) that provides the minimum total error, which does not exceed one increment (the least-significant bit of a Y-register) during even a very long interval of integration. The simplest but widespread case of one and two IDI-system was considered and the general expression was obtained for the static correction of the total error. For the single IDI the linear dependence between \( R_0 \) and an initial condition (\( Y_0 \)) of sub-integral function was found. Moreover
the zone with the upper and lower boundaries of $R_0$ and also the $R_0$-optimal with minimum RMSE were located. Studies were also made for a two-IDI system; the similar linear dependences, the upper and lower boundaries of $R_0$ and the $R_0$-optimal with minimum RMS error were obtained. In conclusion, the method shows the robust behavior arising from selecting the $R_0$ from the wide range of the initial values.

References:

Эффективность управления скоростью индукционной машины (ИМ – асинхронной машины) преимущественно зависит от точности измерения текущего значения её магнитного потока. Данная статья развивает проблему, связанную с квази-идеальным цифровым интегратором в форме современного цифрового дифференциального анализатора (ЦДА) – инкрементного цифрового интегратора ИЦИ. Предложенный метод компенсирует общую погрешность, возникающую собственно в "чистом" интеграторе и в результате ошибки измерения. В работе подробно рассматривается разработка метода полного исправления (коррекции) погрешностей ИЦИ типа цифрового дифференциального анализатора. ИЦИ это – элемент для обработки цифровых сигналов ввода-вывода, используя принцип ЦДА. В первую очередь представлены основные типы погрешностей ИЦИ, а также исследованы причины их появления. Исследовано типовое дифференциальное уравнение dY=aYdx, как пример для анализа количественного соотношения погрешностей. Погрешность ИЦИ по сравнению с аналитическим решением не превышает одного кванта подинтегральной функции Y даже для очень длинного интервала интегрирования переменной x. Это означает, что ИЦИ становится фактически идеальным интегратором. Предложенные методы коррекции погрешностей ИЦИ могут быть применены в имитации, моделировании, и особенно для управления динамическими системами, и т.д. Этот метод легко применим в устройствах управления скоростью индукционной машины основанных на косвенном способе измерении магнитного потока машины.

Ключевые слова: индукционная машина, цифровой интегратор, коррекция ошибок.

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