UDC 519.6

O. O. Sytnyk, Ph. D., Professor
Cherkasy State Technical University, Cherkasy

ANALYTICAL METHOD OF FORMING INTEGRATED DYNAMIC MODELS AND THEIR SOFTWARE IMPLEMENTATION

In represented work the integral method of dynamic objects simulation is considered. It is shown, that the offered method has a large generality and allows to decide on wide scientific fields of problems of mathematical simulation, which can be described with the integral equations. It is marked, that the offered approach allows to use new numerical algorithms, the number of traditional problems solving by means of computer aided simulation. The universal software for the solution of integral equations in MATLAB is developed.

Key words: *dynamic models, equivalent forms, integral equations, numerical algorithms, computer implementation.*

Intensive development of simulation methods is based on the use of huge possibilities of modem computers. However the computer application is possible only after fulfillment of large volume of work concerned with the mathematical description of a soluble problem, search of a successful mathematical model adequately mapping the real process and at the same time accessible for research and obtaining of quantitative results. At this stage the problem of using other types of equations or other mathematical relations is solved. The ultimate methodology of mathematical simulation is oriented on application of the finite equations, ordinary differential equations or equations represented in the form of partial equations having already become the traditional mathematical models for many physical phenomena. So, for describing the processes and conditions in objects with the parameters, distributed in space, the differential equations represented in the form of partial equations are used, for describing the processes in objects with lumped parameters — ordinary differential equations are used.

Recommendations for applications of means of integral equations exist in less quantity, though the amount of their applications will continuously increase. There are the problems, for description of which it is principally impossible to apply other approach, except for using the integral equations. The application of integral equations allows to reduce the dimension of some research problems of continuous mediums, to formulate boundary value problems more compactly, than the differential equations, leads to the stable computing procedures [1]. The use of integral equations as the means of investigation of physical objects and processes leads to forming the independent approach to simulating based on the methods totality of determination of correlations between the known initial data and difiniendum characteristics of the investigated phenomenon, and also the methods of equivalent transformations of the obtained integral equations and their exact or approximate solution.

Let's mark, that practically any form of a mathematical model description can be reduced to an integral equation by equivalent transformations (property of universality [2]), though the inverse transformation in a common case is impossible. The numerical algorithms of integral equations solutions are original' and more often have no analogs among algorithms of solutions other equations types, which are equivalent on the mathematical statement. The equivalent transformations of the formulas and equations represent a fundamental, powerful method of deriving both theoretical, and practical results in mathematical simulation. The ambiguity of a problem of choice and construction of model of some real system stipulates the high expediency of effective application of equivalent transformations of any of the initial mathematical description with the purpose of obtaining the more convenient model from the point of view of realization. Obtaining a number of models with their consequent comparative evaluation is possible.

Let's consider singularities of the description and research of typical problems of systems dynamics on the base of the application of one-dimensional integral dynamic models [2].

The integral equations can be obtained immediately according to a structure of a dynamic system on impulse transfer and nonlinear characteristics of elements, or by equivalent or approximate transformations of the differential or integral-differential equations noted on the base of physical laws.

The traditional form of the description of one-dimensional dynamic objects is the description in the form of the ordinary differential equations with the given initial conditions (Cache problem). Let's remark, that such form of the description in its nature is closely connected to the integral equation of Volterra type of the second kind. The solution of the ordinary differential equations system may be analytically noted in the form of integral Volterra equation of the first kind through a fundamental matrix. However such method of description with the integral equations practically is not used in mathematical simulation, as leads to necessity of fulfillment of extremely complicated calculations.

Sparse methods of equivalent transition from the ordinary differential equations to the integral ones is used usually for illustration of some theoretical aspects, however these methods deserve attention from an applied point of view.

The most common method of transition to integral equations is the method of the analytical inversion with decomposition. A model of object as the ordinary differential equation let is known

$$D[y] \equiv y^{(n)} + \sum_{i=1}^{n} a_i y^{(n-i)}(t) = f(t), \tag{1}$$

$$y^{(i)}(0) = C_i,\ i = \overline{0, n-1}.$$

After decomposition integral operator $D[y]$ on two ones, we shall re-write the equation (1) in the following form

$$y^{(n)}(t) + \sum_{i=1}^{m} a_i y^{(n-i)}(t) = f(t) - \sum_{i=m+1}^{n} a_i y^{(n-i)}(t). \tag{2}$$

After a change of variables $u(t) = y^{(n-m)}(t)$, $u'(t) = y^{(n-m+1)}(t)$, …, $u^{(m)}(t) = y^{(n)}(t)$ we obtain the equation of m order

$$u^{(m)}(t) + \sum_{i=1}^{m} a_i u^{(n-i)}(t) = \varphi(t), \tag{3}$$

where

$$\varphi(t) = f(t) - \sum_{i=m+1}^{n} a_i u^{(n-i)}(t). \tag{4}$$

Going from (2) to the equivalent system of the differential equations and using the fundamental solutions in connection with a canonical system of the differential equations we obtain the integral equation with a kernel of an exponential type. The unknown variables of equation (1) and (3) are connected by dependence

$$y(t) = \int_0^t \ldots \int_0^t u(s)\,ds = \int_0^t \frac{(t-s)^{(n-m-1)}}{(n-m-1)!} u(s)\,ds. \tag{5}$$

The transition from one variant of a model to other one is carried out by a modification of value $m \in \overline{1, n}$.

If in the above considered method to accept $m = n$, the obtained variant of the method is named as the method' of a sequential integration. In this case equation (1) by a sequential n-multiple integration is reduced to the integral equation of the following type

$$y(t) = \int_0^t K(t-s) y(s)\,ds = F(t), \tag{6}$$

where

$$K(t-s) = \sum_{i=1}^{n} q_i \frac{(t-s)^{(i-1)}}{(i-1)!}, \tag{7}$$

$$F(t)=\int_0^t \frac{(t-s)^{(n-1)}}{(n-1)!} f(s)\,ds+\sum_{i=0}^{n-1} C_i \frac{t^i}{i!}+C_0 \sum_{i=1}^{n-1} q_i \frac{t^i}{i!}+ \\ +C_1 \sum_{i=1}^{n-2} q_i \frac{t^{(i+1)}}{(i+1)!}+\ldots+C_{n-2} \frac{t^{(n-1)}}{(n-1)!}. \quad (8)$$

If we accept *m = 0*, the method of a higher derivative is obtained. In this case the replacement $u(t)=y^{(n)}(t)$, $\int_0^t u(s)\,ds=y^{(n-1)}(t)$ and etc. is used which allows to obtain the equivalent integral equation concerning a higher derivative of the initial equation (1):

$$y^{(n)}(t)=\int_0^t \sum_{k=1}^{n} a_k \frac{(t-s)^{(k-1)}}{(k-1)!} y^{(n)}(s)\,ds=\varphi(t), \quad (9)$$

$$\varphi(t)=f(t)-C_{n-1}a_1-(C_{n-1}t+C_{n-2})a_2-\ldots- \\ -\left(C_{n-1}\frac{t^{(n-1)}}{(n-1)!}+\ldots+C_1 t+C_0\right)a_n. \quad (10)$$

As an example we shall consider a solution of the differential equation with constant coefficients [1]

$$y''+y'+y=3e^{-2t};\ y(0)=1,\ y'(0)=-2$$

by reduction to an equivalent integral equation by a method of a higher derivative and method of a sequential integration. Solutions of this test example are indicated on fig. 1.

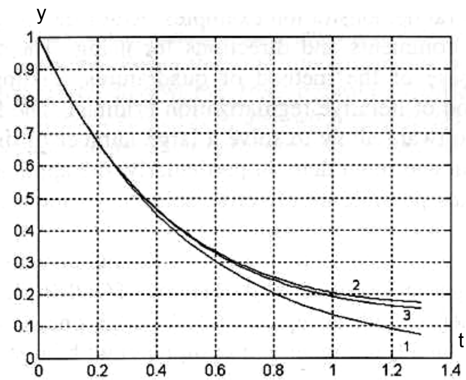

Fig. 1.

Here: 1 — exact solution; 2 — solution of an equivalent integral equation, obtained by a method of a higher derivative with a step h = 0.1:

3 — solution of an equivalent integral equation, obtained by a method of sequential integration, with a step $h = 0.1$.

Let's mark, that the method of a sequential integration is more preferable from the point of view of numerical realization. This method assumes preliminary n integration of function $f(t)$, which is usually known approximately, as the result of measurement. The integration operation allows considerably to reduce influence of unbiased errors of a measurement upon the accuracy of solution of integral equation.

Let's mark, that the method of a sequential integration is more preferable from the point of view of computer realization. This method assumes a preliminary n-multiple integration of function $f(t)$, which is known approximately, as a data of a measurements. The operation of an integration allows considerably to reduce influence of unbiased errors of a measurement by an exactitude of an integral equation solution. The accuracy of the delivered problem solution can be considerably increased decreasing an integration step.

The integral operators and equations with constant limits of integration are used for the description of various types of boundary value problems. As well as in case of the initial problem, the integral dynamic models describing the boundary value problems, have a high level of universality. All of the differential equations with limits as boundary conditions are reduced by equivalent transformations to the above mentioned dynamic models. The integral equations with constant limits of integration connect the given and desired functions on the finite interval, that is the most naturally corresponds to statement of boundary value problem, and the differential equations determine this connection on infinitesimal small interval with taking into account the additional conditions.

One of known methods of obtaining the explicit model of a linear boundary value problem [3] is based on the application of Green function. It is also possible to use a Laplace transformation of the initial differential equations in partial derivatives for construction of mathematical models of objects with the distributed parameters [2], Obtained such a way transfer functions of objects contain fractional exponents of variable p. In this case the ordinary differential equation with partial derivatives corresponds to the simulated object. At simulating in time area it is enough to find and to realize an appropriate integral operator of the fractional order.

For realizing the integral mathematical models by means of computer aided simulation the universal software — INTEGRAL EQUATION TOOLBOX functioning in MATLAB is developed [4; 5]. The absence of effective universal software intended for a solution of various integral equations, taking place at solving some of the important applied problems, originating for want of a solution, (analysis of processes in dynamic systems, determination of impulse function of linear systems, problem of optimum filtration, problem restoring of signal etc.) was premise for creation of the given software.

The software is intended for solution of integral Volterra equations of the first and second kind, the Fredholm equation of the first, second and third kind and also integral Volterra and Fredholm equations of the first kind such as a convolution. The software consists of sixteen main programs, ten service subprograms and several demonstration examples. All of the programs are supplied with the detailed comments and directions for using. The programs are constructed on the base of the method of quadratures, Tikhonov regularization method and method of iterative regularization Fridman. The flexibility and universality of this software allow to solve a large number of different variants of problems statement and input data. In particularly, the application of regularization methods makes possible an effective solution of integral equations of the first kind relating to ill posed problems.

The software is created on the basis of IBM PC in WINDOWS with using MATLAB language. INTEGRAL EQUATIONS TOOLBOX have the structure in the correspondence with accepted in MATLAB one, can be connected to MATLAB of version 4.0 or higher and is supplied with large number of demonstration examples.

References

1. Verlan A. F. Integral Equations: methods, algorithms, programs / A. F. Verlan, V. S. Sizikov. — K. : Naukova dumka,1986. — 544 p. (in Russian).
2. Verlan A. F. Mathematical simulation of continuous systems / A. F. Verlan, S. S. Moskalyuk. — K. : Naukova dumka, 1988. — 288 p. (in Russian).
3. Krasnov M. L. Integral Equations / M. L. Krasnov, A. I. Kiselev, G. I. Makarenko. — 2^{nd} ed. — M. : Nauka 1976. — 216 p. (in Russian).
4. INTEGRAL EQUATIONS TOOLBOX — software package for a solution of integral equations in MATLAB. Institute of Simulation Problems in Power Engineering of National Science Academy of Ukraine / A. F. Verlan, D. E. Kontreras, S. V. Sizikov and other. — K., 1997. — 44 p. (in Russian).
5. Verlan A. F. INTEGRAL EQUATIONS TOOLBOX — programs set for solution of integral equations in MATLAB / A. F. Verlan, D. E. Kontreras, S. T. Tikhonchuk // Systemy Komputerowe i Sieci. Proektowanie, zastosowanie, eksploatacja, Material Midzynarodowej Konferencji Naukowej. — Rzeszow, 1997. — P. 263–269.

У представленій роботі розглядається інтегральний метод динамічного моделювання об'єктів. Запропонований метод має велике значення і дозволяє розв'язувати широкий клас задач математичного моделювання, які можуть бути описані інтегральними рівняннями. Цей підхід дозволяє використовувати нові чисельні алгоритми для вирішення багатьох традиційних проблем за допомогою комп'ютерного моделювання. Розроблене універсальне програмне забезпечення для розв'язання інтегральних рівнянь в MATLAB.

Ключові слова: *динамічні моделі, еквівалентні форми, інтегральні рівняння, чисельні алгоритми, комп'ютерна реалізація.*

Отримано: 20.09.2012