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APPROACH TO ENERGY OBJECTS' DYNAMICS MODELLING BASED ON SINGULAR SYSTEMS' ELEMENTS

Over the last decades there has been substantial progress on the development of theory and numerical methods for singular systems (known also as descriptor systems, semistate systems, differential algebraic systems, generalized state-space systems, etc.). The need for such methods arisen primarily from the increased practical interest for a more general system description which takes the intrinsic physical system model structure into account. Besides that, many physical processes are most naturally and easily modelled as mixed systems of differential and algebraic equations (DAE). As the title implies the paper describes the singular systems theory application in power systems dynamics simulation, particularly considered an alternative method for energy systems' mathematical models formulation based on the singular systems theory elements with some indicative examples illustrating feasibility and efficiency of this approach.

Keywords: differential algebraic equations, singular systems, energy systems dynamics modelling.

Introduction. An energy object (or system) is a complex one consisting of a large number of units of energy production interconnected by the network of transmission and distribution; the complexity of this system is increasing every day due to the exponential rise in its demand and increased dependence on its reliability and security. Therefore, due to economical and technical reasons, there is an increasing need for enhancement of mathematical models formulation methods, as well as development of new numerical methods and software tools for power systems simulation). Let us consider an alternative method for energy systems mathematical models formulation based on the singular systems elements, with a few examples to illustrate the feasibility and efficiency of this approach.

Fundamentals of singular systems theory for energy systems' dynamics modelling. Over the recent years, considerable efforts have been devoted to the development of theory and numerical methods for singular systems, but it is a reflection of the infancy of the field that a common terminology has not yet involved. Such systems also have been called in literature «implicit systems», «descriptor systems», «semistate systems», «differential algebraic systems», «generalized state-space systems», and perhaps other names. In fact, the term «singular system» (or equivalent) is understood as physical system, process or phenomenon, which mathematical model consists of differential algebraic equations. These systems arise naturally in various fields including robotics [1], optimal control economics [2], large scale interconnected systems, automatic control systems [3], electrical networks [4], dynamics of thermal nuclear reactors [5], etc.

The dynamic behaviour of numerous problems in physics, in chemistry and in engineering applications can be modelled by differential equations. Additionally, the models often contain implicit nonlinear algebraic equations in order to take into account conservation laws, geometrical or kinematic constraints, Kirchhoff's laws, etc. [1-5, 6]. Hence, as it was mentioned above, the singular systems are the physical systems that are naturally modelled by differential algebraic equations.

Differential algebraic equations (DAE) are everywhere singular implicit ordinary differential equations (ODE) [1-6, 7]).

$$F(x', x, t) = 0\,, \qquad (1)$$

where the partial Jacobian $\partial F / \partial x'$ identically singular (is singular for all values of its arguments). If $\partial F / \partial x'$ were non-singular, (1) could be solved for x', at least theoretically, and we would have an explicit ODE. Actually, DAEs are ODEs but those which cannot be solved with respect to the x' [6].

An important feature of such systems is that the equations (1) may have no solution, a unique solution or an infinite number of solutions, each of which is not necessarily isolated. Therefore, in contrast to explicit ordinary differential equation, the integration of DAEs may cause essential difficulties [4-7].

Implicit models advantages in energy systems dynamics modelling. The efficiency of a model format depends on the type of problem to be solved, but there are a few following features of implicit models which justify they use for modelling energy systems dynamics:

1) *generality:* implicit models (1), due to their specific features, describe a more general class of problems than ordinary differential equations models. Naturally, DAEs allows to take the intrinsic physical system model structure into account;
2) *feasibility:* DAE can be represented in a computer (at specific time point of solution) as matrices of constant real numbers; it is cause a convenience of computer manipulations with such models;
3) *naturalness:* as mentioned above, DAE systems arise naturally in various fields when differential equations of dynamic behaviour complete with different algebraic (or transcendental) constraints, such as conser-

vation laws, geometrical or kinematic constraints, Kirchhoff's laws, etc. DAE also arise when discretizing spatial operators during the solution of partial differential equations;

4) *uniqueness:* DAE models usually apply to problems, which cannot be modelled by ODE due to their specific features;
5) *simplicity:* the same way as most of special-purpose methods using the DAEs method utilization resulting in the best way (in a certain sense) organized model. Besides that, implicit representation can give a conceptual simplicity to a number of specific features of investigated problem, consider a few examples of singular power systems.

Example 1.1. The dynamic behaviour of a power circuit is described by a set of nonlinear ordinary differential equations and a set of nonlinear algebraic equations as [8, 9]:

$$dx/dt = f(x, V)\,, \tag{2}$$

$$I(x, V) = Y \cdot V\,, \tag{3}$$

where x is the state vector, f is nonlinear vector function, V is the vector of complex nodal voltages, Y is the nodal admittance matrix and I is the vector of injected nodal currents.

The differential equations represent the dynamic behaviour of the system elements (transformers, induction motors, synchronous machines and their controllers, power electronics, etc.) and the algebraic equations represent the network equations and the connection of the external elements to the network.

Example 1.2. Large thermal nuclear power reactors can exhibit unstable or underdamped oscillation in the power distribution, with periods of 30-40 hours, due to the effects of the fission product poison xenon-135. The dynamics of such systems can be approximated by coupled algebraic and differential equations of the form [5]:

$$\begin{aligned} \dot{x}(t) &= A\,x(t) + B\,u(t), \\ E\,u(t) &+ F\,x(t) = 0, \end{aligned} \tag{4}$$

where A, B, E, F are real matrices; $x(t)$ represents the internal states inherent in the system, namely, xenon and its precursor iodine, and the power distribution and control rod reactivity are lumped together in $u(t)$.

Example 1.3. The neutron transport equation in nonstationary case is of the form [10]:

$$\begin{aligned} &\frac{\partial \psi(t, x, \mu)}{\partial t} + \mu\,\frac{\partial \psi(t, x, \mu)}{\partial x} + \sigma\,\psi(t, x, \mu) = \\ &= \frac{\sigma_s}{2}\int_{-1}^{1} \psi(t, x, \mu')\,d\mu', \quad 0 \le t \le T < \infty, \quad 0 \le x \le 1, \end{aligned} \tag{5}$$

$$\psi(t, 0, \mu) = 0, \quad 0 \le \mu \le 1,$$
$$\psi(t, 1, \mu) = 0, \quad -1 \le \mu \le 0,$$
$$\psi(0, x, \mu) = f(x, \mu).$$

By choosing a sequence of $-1 \le \mu_0 < \mu_1 < \dots < \mu_N \le 1$ and applying any quadrature rule to the right part of the formula (5) we obtain the next system of equations

$$\frac{\partial \psi_i(t, x)}{\partial t} + \mu_i \frac{\partial \psi_i(t, x)}{\partial x} + \sigma \psi_i(t, x) = \sum_{j=0}^{N} C_j \psi_j(t, x) \; i = 0, 1, \dots, N. \quad (6)$$

By applying a method of weak approximation [11] to the system [12] we obtain

$$\frac{\psi_i^{n+1/2}(x) - \psi_i^n(x)}{\tau} = \sigma \psi_i^{n+1/2}(x) + \sum_{j=0}^{N} C_j \psi_j^{n+1/2}(x),$$
$$\frac{\psi_i^{n+1}(x) - \psi_i^n(x)}{\tau} + \mu_i \frac{d \psi_i^{n+1}(x)}{d x} = 0, \quad (7)$$
$$i = 0, 1, \cdots, N; \quad n = 0, 1, \cdots, M; \quad \tau = \frac{T}{M}; \quad 0 \le x \le 1.$$

Here $\psi_i^0(x) = f(x, \mu_i)$, $\varphi_i^n(0) = 0$ (when $0 \le \mu_i \le 1$) and $\varphi_i^n(1) = 0$ (when $-1 \le \mu_i < 0$).

Equations (7) constitute a system of differential algebraic equations relative to unknown functions $\psi_i^n(x)$, $i = 1, \cdots, M$ and $\psi_i^{n+1/2}(x)$, $i = 0, 1, \cdots, M - 1$.

Conclusions. Over the recent years there is a need in various fields of energy applications for a more general system description which takes the intrinsic physical system model structure into account. Therefore, there is an increasing demand for alternative methods for energy systems' mathematical models formulation. The efficiency of a model format depends on the type of problem to be solved, but there are a few reasons for singular models using in modelling energy systems' dynamics. They are generality, feasibility, naturalness, uniqueness and simplicity of this approach. There is a variety of problems in energy systems dynamics' modelling which are described by DAE. Among them are the power circuit dynamics simulation problem, the dynamics of thermal nuclear reactor modelling problem and the neutron transport equation solution problem highlighted in this paper.

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ПІДХІД ДО МОДЕЛЮВАННЯ ДИНАМІКИ ЕНЕРГООБ'ЄКТІВ НА ОСНОВІ ЕЛЕМЕНТІВ СИНГУЛЯРНИХ СИСТЕМ

За останні десятиліття відбувся значний прогрес у розвитку теорії та чисельних методів для сингулярних систем (відомих також як системи дескрипторів, системи напівстанів, диференціальні алгебраїчні системи, узагальнені системи простору станів тощо). Потреба в таких методах виникла в першу чергу через підвищений практичний інтерес до більш загального опису системи, який бере до уваги структуру внутрішньої фізичної моделі системи. Крім того, багато фізичних процесів найбільш природно і легко моделюються у вигляді змішаних систем диференціальних і алгебраїчних рівнянь (ДАУ). Як випливає з назви, стаття описує застосування теорії сингулярних систем у моделюванні динаміки енергетичних систем, зокрема, розглядається як альтернативний метод формулювання математичних моделей енергетичних систем на основі елементів теорії сингулярних систем з деякими показовими прикладами, що ілюструють здійсненність та ефективність цього підходу.

Ключові слова: *диференціально-алгебраїчні рівняння, сингулярні системи, моделювання динаміки енергетичних систем.*

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RESEARCH OF GEOMETRIC AND INFORMATION MODELS FOR AWNING STRUCTURES

The article analyzes membrane (awning) structures, which become relevant due to their cost-effectiveness and the creation of original forms. The characteristics of awning structures, the possibilities of molding, the use of various materials and combined options for combining an awning with other materials are considered. Due to their cost-effectiveness, tent structures are becoming increasingly popular today, because in modern socio-economic conditions there is a need for the rapid construction of low-cost buildings to overcome the shortage of mobile housing and structures for other purposes. In the conditions of restoration of the lost objects of buildings and structures, the use of tent coverings is important. Their development was held back for a long time due to the non-compliance of domestic tent materials with the high requirements for tent coverings of this type, namely: strength, durability, color diversity, light fastness, etc. The use of hinged structures allows you to create small architectural forms and mobile buildings that are not only quickly erected, but also easily transformed in accordance with a change in functionality. This allows you to create new types of objects, such as stadiums, airports, giant greenhouses, botanical gardens, warehouses, etc. Modern technologies combine the advantages of industrial construction methods with the individualization of form and open the way to the use of various awning structures. Membrane coatings, as one of the modern trends in the presentation of a new form of roofing, create new spatial characteristics of architectural objects. They form expanses freed from bulky internal structures. The freedom of space determines the flexibility and functionality of its use, a high degree of adaptation and, as a result, the durability of the

