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ON THE CONTROL OF NUMERICAL RESULTS IN THE PROBLEMS OF IDENTIFICATION OF DYNAMIC ENERGY OBJECTS

The article proposes an approach to solving actual problem of the obtained results' control in the construction and implementation of algorithms for identifying energy objects based on integral dynamic models. The considered method based on the use of quadrature algorithms and splines for approximation of the kernel, with the transition to solving equations with a degenerate kernel using recurrent formulas, showed sufficiently high efficiency in solving the problems of obtaining high performance at provided control over the accuracy of the integral dynamic model parameters' calculation, as well as providing resistance to the experimental data errors. The proposed method makes it possible to solve the problem of accumulating calculations, which, in turn, leads the numerical implementation algorithm to a form in which it is feasible to obtain solutions in real-time. The obtained integrated models have a sufficient level of adequacy and can be used in integrated computing control systems for energy objects.

Keywords: *Integral models, degenerate kernel, model identification, control of identification results.*

Introduction. Power energy objects are highly stressed engineering systems of high complexity, which use advanced scientific achievements and realize new capabilities of modern technology. A complete mathematical description of such engineering objects is achieved by complex systems of partial differential equations with rather difficult boundary conditions.

The solution of such systems of equations is quite time-consuming and requires significant computer time, which limits the comprehensive analysis of the object and in practice is reduced to use only for the initial study of the object's dynamic properties, as well as for the final verification of the obtained laws and algorithms for modelling and control (and this is in case that it is generally possible to find stable methods of numerical solution of such systems, which, as practice shows, is not always possible to achieve) [1]. An effective method of overcoming these problems is use of non-parametric models of dynamic objects in the form of integral equations or operators. Since power energy objects are technically complex objects, obtaining their models based on experimental data simplifies the identification problem and provides a possibility to describe objects with concentrated and distributed parameters without changing the structure of the model. Such a versatility of integral models leads to the use of the same algorithms for both concentrated and distributed objects. However, with the numerical implementation of integral models, there is a problem of accumulating calculations, which, to a large extent, limits their application in practice. Therefore, the problem of finding methods that would not have the effect of accumulating calculations and could provide solutions in real-time remains relevant.

Model approach to solving identification problems. As known [2-4], in the problem of identification, we are dealing with underdefined object-model systems. The difference is that in the problem of identification model is underrecognized (while the object is considered sufficiently studied) and efforts of researchers aimed at finalizing the model [5]. Identification algorithms, except for the least squares method, have an important disadvantage that limits their use — the presence of statistical data and the manipulation of random processes.

Creating formalized models of complex dynamic objects is associated with the compression of descriptive and factual information. The first stage in solving the given problem is identification of the research object — construction of its mathematical model based on the results of experiments. The problem of identification is formulated as follows: according to the results of observations on the input and output variables of the object, build a model that is optimal in some sense. The practical solution to the problem of identification is a computational procedure for evaluating unknown parameters of the mathematical model resulting in establishing regularities of the original object functioning [6, 7].

When solving the problems of dynamic energy objects (DEO) identification, particularly in the construction and implementation of algorithms for the identification of energy objects based on integral dynamic models [8, 9], the actual problem is control of the results obtained, meaning the solution of the following tasks:

- providing high-performance control of the accuracy of the integral dynamic model parameters' calculation, as well as resistance to errors in experimental data;
- reducing the numerical implementation algorithm to a form that eliminates the problem of accumulating calculations, which provides real-time functioning.

Method description. Method of assessing the built DEO model consists in its numerical implementation and comparison of the results obtained with the original data. Thus, the validation phase is a direct task of analyzing models of a dynamic object. As criteria for the parameters' calculation accuracy can be accepted the following: relative error, difference module, standard deviation of the model output signal from the measured at the object, etc. Consider a case when the DEO model output signal is an integral equation solution. In this regard, we consider some algorithms for the numerical solution of Volterra integral equations which is advisable to assess the accuracy of solving the identification problem.

Quadrature algorithms. Consider quadrature algorithms for solving Volterra equations of the II kind with a difference kernel of the form:

$$y(t)+\int_{0}^{t} K(t-s)y(s)ds = F(t)\,, \qquad (1)$$

which describe stationary objects with both concentrated and distributed parameters.

The basis of the numerical implementation of integral operators and equations is replacement of integrals with finite sums [10, 11]. In this case, various quadrature formulas with significant algorithmic features can be applied. For a wide class of equations, the method of quadrature formulas in the case of using left and middle rectangles, as well as trapezoids, leads to effective algorithms. The traditional approximation algebraic expression for the integral equation (1) has the form

$$y(t_i)-\sum_{j=1}^{i} A_j K(t_i-t_j) \cong F(t_i)\,, \qquad (2)$$

where $j=1, 2,...i$, $(i=\overline{1,n})$ — discretization nodes, A_i — quadrature formula coefficients. If the points t_i follow each other in increments $A_i\,t_i$, $h=h^i=\text{const}$, then $t_i=(i-1)h$. In accordance with dependencies (2), it is possible to write a calculation expression to find the approximate discrete values of the desired function $y(t)$:

$$y_1 = F_1,$$
$$y_i = \frac{1}{1 - A_i K_{ii}} (F_i + \sum_{j=1}^{i-1} A_j K_{ij} Y_j), \qquad (3)$$

where $y_i = y(t_i), K_{ij} = K(t_i - t_j), f_i = f(t_i), 1 - A_i K_{ii} \neq 0.$

From the expression (3) as the step number of the sampling nodes increases, the number of operations performed at each step of the calculations increases, and consequently, increases the amount of memory required and the solution time for computer calculations. The difficulties of the numerical solution of the considered equation are determined to a large extent by the type of kernel. The above difficulties can be largely overcome if the kernels are separable (degenerate), i.e.

$$K(t - s) = \sum_{k=1}^{n} \alpha_l^k(t) \beta_l^k(s),$$
$$l = \overline{0, n-1}, \qquad (4)$$

where, in particular,

$$\alpha_l^k(t) = \frac{1}{(k-1)!} C_{k-1}^l t^{k-1-l}, \beta_l^k(s) = (-1)^l s^l ,$$
$$k = \overline{1, n}, l = \overline{0, k-1} .$$

In our case, the kernel is separable and given (4), the expression (2) takes the form

$$y(t) = \sum_{k=1}^{n} \alpha_l^k(t) \int_0^t \beta_l^k(s) y(s) ds + F(t) . \qquad (5)$$

Using quadrature formulas, we obtain recurrence expressions for solving equation (5):

$$y_1 = F_1,$$
$$y_i = \frac{1}{1 - \sum_{l=1}^{m} \alpha_{li} \beta_{li} A_i} \left(f_i + \sum_{l=1}^{m} \alpha_{li} \sum_{j=1}^{i-1} A_j B_{lj} Y_j \right), \qquad (6)$$

where $\alpha_{li} = \alpha_l(t_i), \beta_{li} = \beta_l(t_i).$

Expression (6) is an algorithm that differs from (3) in that the number of calculations at each step remains unchanged.

Application of the trapezoidal formula with a constant step $h = h_i = \text{const}$ in the expression (6) gives the following calculation formulas:

$$y(0) = f(0),$$

$$y_i = \frac{1}{1 - h\sum_{l=1}^{m} A_i \alpha_{li} \beta_{li}} (f_i + h\sum_{l=1}^{m} \alpha_{li} \sum_{j=1}^{i-1} A_j \beta_{li}, y_i),, \quad i = 2, 3, ..., n, \qquad (7)$$

where $A_j = \begin{cases} 0,5, \text{ when } j = 1 \text{ and } j = n \\ 1, \text{ when } j > 1 \end{cases}.$

To control the accuracy of the DEO dynamic models' parameters, the following computational experiment was performed on the based on the above algorithm.

We assume that the input and output signals are specified analytically and, in addition, the initial conditions and order of the model are known. The initial data are shown in Table 1.

Table 1

T	*F*(*t*)	*y*(*t*)	C_0	C_1	C_2	C_3	C_4	C_5
1	6 sin*t* + cos*t*	sin*t*	0	—	—	—	—	—
2	3 sin*t* + 2 cos*t*	sin*t*	0	1	—	—	—	—
3	$e^{-t}\cos t(10t - 11) + e^{-t}\sin t(9t - 2)$	$te^{-t}\cos t$	0	1	– 2	—	—	—
4	$120(t^5 + t^4 + t^3 + t^2 + t)$	t^5	0	0	0	0	—	—
5	$720(t^6 + t^5 + t^4 + t^3 + t^2 + t)$	t^6	0	0	0	0	0	—
6	$5040\,(t^7 + t^6 + t^5 + t^4 + t^3 + t^2 + t)$	t^7	0	0	0	0	0	0

The results of the calculation are given in Table. 2 and indicate the acceptable accuracy of the results obtained in the calculation of the dynamic model parameters. In Fig. 1. a graph of the residual module function (in conventional units) against time is presented.

Table 2

t	y	$\tilde{y}$	$\lvert y - \tilde{y} \rvert$
0	0	0	0
0.1	0.09983	0.09871	0.00112
0.2	0.198669	0.196227	0.00244
0.3	0.29552	0.29341	0.00211
0.4	0.38941	0.38433	0.00508
0.5	0.47942	0.47215	0.00727
0.6	0.56464	0.55684	0.0072
0.7	0.644217	0.63532	0.00889
0.8	0.717356	0.70265	0.014706
0.9	0.783226	0.77124	0.011986
1	0.841470	0.82833	0.01314

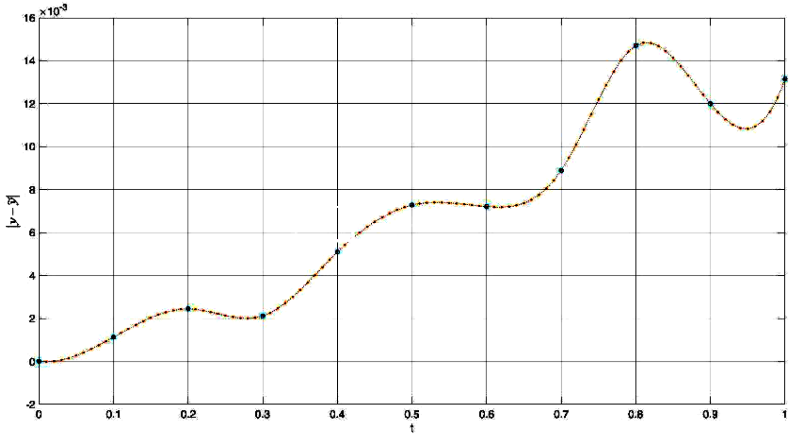

Fig. 1. *Plot of the function of the residual modulus against time*

Spline method. Let us consider a way to implement integral dynamic models with an arbitrary kernel by using interpolation cubic splines to approximate the function $K(t)$, defined by its values $K_l = K(t_l), l = \overline{0, N}$ on the segment [0, *T*] [12]. The need for such an implementation arises for a stationary DEO, when, after restoring the model parameters, the values of the kernel (apparatus function) are determined at the measurement points.

For this purpose, you can use an approximating expression

$$y(t) + \int_0^t \left[q_1 + q_2 (t - s) \right] y(s) ds = \int_0^t (t - s) f(s) ds + c_0 + c_1 t + q_1 c_0 t,$$

which we represent in the form of:

$$\tilde{K}(t) = d_{1l}(t_l - t)^3 + d_{2l}(t - t_{l-1})^3 + d_{3l}(t_l - t) + d_{4l}(t - t_{l-1}), \tag{8}$$

$$d_{1l} = \frac{m_{l-1}}{6h_l}, d_{2l} = \frac{m_l}{6h_l}, d_{3l} = \frac{K_{l-1} - \frac{m_{l-1}h_l^2}{6}}{h},$$
$$d_{4l} = \frac{K_l - \frac{m_l h_l^2}{6}}{h_l}, t_{l-1} \le t \le t_l, \quad l = \overline{1, N}. \tag{9}$$

Substituting (8) into the original equation (1), at $x = x_i (1 = \overline{1, N})$, we obtain:

$$y(t_i) + \sum_{l=1}^{i} \int_{t_{l-1}}^{t_l} \{ d_{1l}[(t_l - t_i) + s]^3 + d_{2l}[(t_i - t_{l-1}) - s]^3 + \\ + d_{3l}[(t_l - t_i) + s] + d_{4l}[(t_i - t_{l-1}) - s] \} y(s) ds = f(t_i). \tag{10}$$

As can be seen from (10), the use of splines for approximating the kernel makes it possible to proceed to solving an equation with a degenerate kernel. Using the separability properties of the kernel, we transform expression (10) into the form:

$$y(t_i)+\sum_{l=1}^{i}\{d_{1l}[(t_l-t_i)^3\int_{t_{l-1}}^{t_l}y(s)ds+3(t_l-t)^2\int_{t_{l-1}}^{t_l}sy(s)ds+$$
$$+3(t_l-t_i)\int_{t_{l-1}}^{t_l}s^2y(s)ds+\int_{t_{l-1}}^{t_l}s^3y(s)ds]+d_{2l}[(t_i-t_{l-1})^3\int_{t_{l-1}}^{t_l}y(s)ds-$$
$$-3(t_i-t_{l-1})^2\int_{t_{l-1}}^{t_l}s^3y(s)ds]+d_{3l}[(t_l-t_i)\int_{t_{l-1}}^{t_l}y(s)ds+\int_{t_{l-1}}^{t_l}sy(s)ds]+ \quad (11)$$
$$+d_{4l}[(t_i-t_{l-1})\int_{t_{l-1}}^{t_l}y(s)ds-\int_{t_{l-1}}^{t_l}sy(s)ds.$$

After replacing the integral in expression (11) with a quadrature formula, we obtain calculation expressions for determining the values of the sought function at the measurement points: $y_0 = f_0$,

$$y_i=\frac{h}{B_{i+1}}(\frac{f_i}{h}-\sum_{l=1}^{i}\left\{\alpha_{1l}[(t_l-t_i)^3\sum_{t_j=t_{l-1}}^{t_l}A_jy_j+3(t_l-t_i)^2\sum_{t_j=t_{l-1}}^{t_l}A_jt_jy_j+\right.$$
$$+3(t_l-t_i)\sum_{t_j=t_{l-1}}^{t_l}A_jt_j^2y_j+\sum_{t_j=t_{l-1}}^{t_l}A_jt_j^3]+d_{2l}[t_i-t_{l-1}]^3\sum_{t_j=t_{l-1}}^{t_l}A_jY_j-$$
$$-3(t_i-t_{l-1})^2\sum_{t_j=t_{l-1}}^{t_l}A_jy_jt_j+3(t-t_{l-1})\sum_{t_j=t_{l-1}}^{t_l}A_jt_j^2y_j-\sum_{t_j=t_{l-1}}^{t_l}A_jt_j^3y_j]+ \quad (12)$$
$$+d_{3l}[(t_l-t_i)\sum_{t_j=t_{l-1}}^{t_l}A_jy_j+\sum_{t_j=t_{l-1}}^{t_l}t_jy_j]+d_{4l}[(t_i+t_{l-1})\sum_{t_j=t_{l-1}}^{t_l}A_jy_j-$$
$$\left.-\sum_{t_j=t_{l-1}}^{t_l}A_jt_jy_j]\right\}), \qquad j=\overline{0,i-1},$$

where

$$B_i=A_i[d_{1l}(t_l)^3+d_{2l}(-t_{l-1})^3+d_{3l}t_l-d_{4l}(-t_{l-1})],$$

A_j — quadrature formula coefficients, h — quadrature step.

Characteristic feature of the proposed algorithm is that the number of operations does not increase from step to step, as in the case of direct application of the quadrature method (when the difference kernel is nonseparable). The solution in the considered case is carried out according to recurrent formulas that provide high performance when controlling the accuracy of the integral dynamic model parameters' calculation.

Example. It is necessary to solve the equation

$$y(t) - \int_0^t K(t-s)y(s)ds = t, \quad t \in [0,\ 0.6],$$

for which the kernel is given in Table 3.

Table 3

t	0	0.1	0.2	0.3	0.4	0.5	0.6
$K(t)$	0	0.01	0.20	0.29	0.39	0.48	0.56

These values are obtained based on the function $K(t) = \sin t$, for which the exact solution of the considered example is $y = t + \frac{t^3}{6}$.

In the case under consideration, the grid of nodes of the function $K(t)$ is uniform, h = 0,1 and the number of nodes is l = 11. Let us reduce the equation to be solved to the form (12), considering that $f(t_i) = t_i$. For each l-th interval $[t_{l-1}, t_l]$ its own set of coefficients $d_{1j}, d_{2j}, d_{3j}, d_{4j}$ is calculated using formulas (9). The values m_{l-1}, m_l are determined from the algebraic system. The calculated values of the coefficients $d_{1j}, d_{2j}, d_{3j}, d_{4j}$ are given in Table 4.

Table 4

$[t_{j-1}, t_j]$	J	d_{1j}	d_{2j}	d_{3j}	d_{4j}
0.0-0.1	1	0.000	– 0.376	0.998	1.990
0.1-0.2	2	– 0.376	– 0.481	1.990	2.960
0.2-0.3	3	– 0.481	– 0.653	2.960	3.900
0.3-0.4	4	– 0.653	– 0.799	3.900	4.802
0.4-0.5	5	– 0.799	– 0,942	4.802	5.655
0.5-0.6	6	– 0.942	– 1.074	5.655	6.452

The results of calculations using the quadrature formula with a constant step h = 0.05 are shown in Table 5, and the graph of the function of the discrepancy modulus against time in fig. 2.

Table 5

t	y	$\tilde{y}$	$\lvert y - \tilde{y} \rvert$
0.00	0.000	0.000	0.000
0.05	0.050	0.050	0.000
0.10	0.100	0.099	0.001
0.15	0.150	0.148	0.002

Continuation of Table 5

0.20	0.201	0.198	0.003
0.25	0.252	0.247	0.005
0.30	0.304	0.298	0.006
0.35	0.357	0.350	0.007
0.40	0.411	0.403	0.008
0.45	0.465	0.459	0.006
0.50	0.521	0.517	0.004
0.55	0.577	0.577	0.000
0.60	0.636	0.641	0.005

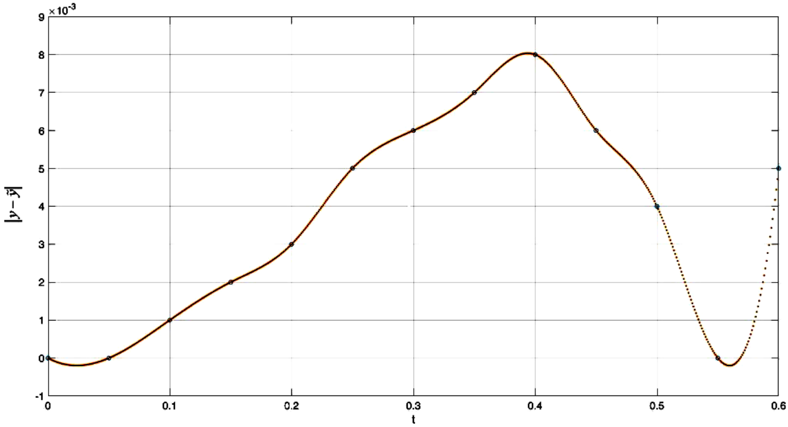

Fig. 2. *Plot of the function of the discrepancy modulus in time*

Conclusions. Thus, the considered algorithm provides an opportunity to assess the accuracy of the dynamic object models' parameters and make a conclusion about the adequacy of the model. The proposed method for quick solution of integral equations, based on using cubic spline approximation of the kernel, shows a few useful features for digital implementation, particularly small and constant number of operations.

The considered algorithms confirm the above theoretical conclusions, are quite effective in terms of the accuracy of the integral dynamic model parameters' and resistance to experimental data errors, eliminate the problem of accumulating calculations, provide real-time operation, and can be proposed for the implementation of algorithms for identifying energy objects based on integral dynamic models.

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ПРО КОНТРОЛЬ ЧИСЛОВИХ РЕЗУЛЬТАТІВ В ЗАДАЧАХ ІДЕНТИФІКАЦІЇ ДИНАМІЧНИХ ОБ'ЄКТІВ ЕНЕРГЕТИЧНОГО ПРИЗНАЧЕННЯ

У статті запропоновано підхід до вирішення актуальної задачі контролю отриманих результатів при побудові та реалізації алгоритмів ідентифікації енергетичних об'єктів на основі інтегральних динамічних моделей. Розглянутий метод, заснований на використанні алгоритмів квадратури і сплайнів для апроксимації ядра, з переходом до розв’язування рівнянь з виродженим ядром за рекурентними формулами, показав достатню ефективність при розв’язуванні задачі отримання високої швидкодії при наявності контролю точності розрахунку параметрів інтегральної динамічної моделі, а також забезпечення стійкості до похибок експериментальних даних. Запропонований ме-

тод дозволяє вирішити проблему накопичення обчислень, що, в свою чергу, приводить алгоритм чисельної реалізації до такого вигляду, при якому можливе отримання розв'язків в режимі реального часу. Отримані інтегральні моделі володіють достатнім рівнем адекватності та можуть використовуватись в інтегрованих обчислювальних системах керування об'єктів енергетичного призначення.

Ключові слова: *інтегральні моделі, вироджене ядро, ідентифікація моделі, контроль результатів ідентифікації.*

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ДОСЛІДЖЕННЯ ВЛАСТИВОСТЕЙ КОЕФІЦІЄНТІВ ДИСКРЕТНОГО КОСИНУСНОГО ПЕРЕТВОРЕННЯ ЯК ОСНОВА МЕТОДА ВИЯВЛЕННЯ ПОРУШЕННЯ ЦІЛІСНОСТІ ЦИФРОВОГО ЗОБРАЖЕННЯ

Одним з найпоширеніших представлень інформації сьогодні є цифрові зображення (ЦЗ), несанкціоновані зміни яких можуть приводити до негативних наслідків як для окремої людини, установи, фірми, так і для держави в цілому, що робить задачу виявлення порушення цілісності ЦЗ одною з найактуальніших задач інформаційної безпеки. Основним недоліком існуючих експертних методів є їх орієнтованість на виявлення результатів конкретної збурної дії, але на практиці експерт часто не володіє інформацію про конкретику атаки на ЦЗ, при цьому набір його засобів завжди є обмеженим, що може привести до ситуації, коли досліджуване ЦЗ помилково бути визнане оригінальним. Першим «ешелоном оборони» тут повинні бути методи, ефективні незалежно від виду збурної дії — універсальні. На теперішній час в відкритих джерелах представлена дуже незначна кількість таких методів, які не є вільними від недоліків, головним з яких є суттєве зниження ефективності в умовах незначних збурних дій. Метою роботи є розробка теоретичного базису для ефективного універсального методу виявлення порушення цілісності ЦЗ, зокрема, в умовах незначної збурної дії. В ході досягнення мети в роботі: обґрунтована доцільність використання блокового підходу при організації експертизи цілісності ЦЗ; область дискретного косинусного перетворення (ДКП) блоку обрана як область