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ON USING GSSM IN POWER SYSTEMS DYNAMICS' SIMULATION

A broad spectrum of aspects related to the feasibility, principles, and implementation of the generalized state-space model (GSSM) as a mathematical modeling technique for energy systems simulation has been considered. This paper reviews the use of GSSM to address the challenges of adequate modeling of dynamics of modern power systems' dynamics, which are characterized by their hybrid nature, complex switching modes, and nonlinear interactions. The versatility of GSSM is analyzed in comparison to traditional approaches, particularly its capability to integrate continuous and discrete system dynamics into a unified advanced framework. Additionally, the model's ability to capture both linear and nonlinear regimes, its compatibility with contemporary computational tools, and its application across various power systems are discussed in detail. The fundamentals, analytical and numerical considerations of differential-algebraic equations (DAEs) are also examined. The effectiveness of GSSM is demonstrated through a case study involving simulation of a power supply control device dynamics. Computer modeling experiments highlight the advantages of GSSM over traditional methods in terms of accuracy, computational efficiency, and scalability. At the same time, they identify areas where further advancements and improvements are necessary.

Keywords: *power systems, generalized state-space model, differential-algebraic equations, mathematical modelling, system dynamics.*

Introduction. The complexity of modern power systems, including renewable energy generation, motor drives, and advanced power electronics, demands robust and efficient modeling techniques to analyze their dynamics. These systems often exhibit intricate transient responses, nonlinear interactions, and hybrid dynamics that challenge traditional simulation methods. Adequate modeling is essential for optimizing performance, diagnosing faults, and enabling real-time control of these systems [1, 2].

The Generalized State-Space Model (GSSM) has emerged as a powerful tool to address these challenges [3]. Unlike conventional state-space approaches, such as averaged models or linear approximations, GSSM provides a unified framework that integrates both continuous and discrete dynamics [4]. This capability is crucial for representing switching dynamics and nonlinear phenomena in systems like pulse-width modulated (PWM) converters, resonant inverters, and grid-tied renewable energy systems. Its adaptability makes it a preferred choice for simulating power systems’ circuits characterized by rapid state transitions and hybrid operating modes [5].

GSSM distinguishes itself with the following key features:

- unified framework: the model combines differential equations for dynamic elements (e.g., transformers, motors, and controllers) with algebraic constraints representing network equations which enables the accurate representation of both transient and steady-state dynamics [6];
- integrating continuous-time dynamics (managed by differential equations) with discrete events (modeled as state jumps or resets), enabling the modeling of systems with switching actions, such as pulse-width modulation (PWM) converters [6, 7];
- handling nonlinearities inherent in power circuits by providing state-dependent switching, which makes it suitable for applications like resonant inverters and soft-switching converters [7];
- compatibility with modern computational tools: GSSM integrates seamlessly with advanced platforms like MATLAB/Simulink, leveraging their computational capabilities for efficient simulation of large-scale systems [7];
- time-varying and piecewise dynamics: the model is particularly effective for systems exhibiting piecewise linearity, as it combines time-varying parameters and mode-dependent dynamics into a cohesive framework [8];
- high fidelity: by accommodating discontinuities and nonlinearities, GSSM surpasses traditional models in capturing the detailed dynamics of modern power systems [9].

Some of the GSSM applications in power systems are as follows:

- DC-DC converters- to buck, boost, and buck-boost converters, providing insights into their transient and steady-state dynamics under varying loads and control schemes [2];
- grid-connected inverters – to support the analysis of grid synchronization and fault detection in renewable energy systems [15,17];
- soft-switching converters – to capture zero-voltage or zero-current switching conditions, which are challenging for traditional state-space approaches [2, 15].

Recent findings have demonstrated GSSM's versatility in several domains:

- DC-DC converter analysis: GSSM has shown enhanced accuracy in predicting transient responses compared to classical methods [5];

- grid-connected inverters: the model facilitates precise control and fault detection, critical for the stability of renewable energy systems [6, 7];
- power factor correction circuits: GSSM has been applied successfully to improve energy efficiency and address power quality challenges [5].

Comparing to the recent popular data-driven methods such as Artificial Neural Networks (ANNs), GSSM retains a significant edge due to its deterministic foundation and compatibility with engineering principles. Unlike ANN-based models, which often rely on large datasets, GSSM provides transparent insights into system dynamics and ensures adherence to physical laws. However, as power systems grow increasingly complex, ongoing advancements in adaptive algorithms and parallel processing are expected to further enhance GSSM's scalability and performance [9]. Thus, GSSM ability to represent nonlinear, hybrid, and time-varying dynamics with high fidelity makes it still actual tool for advancing energy system modeling [10, 11].

GSSM application for power systems' modelling. The power systems' dynamics is described by a set of nonlinear ordinary differential equations and a set of differential-algebraic equations (DAEs): as [2]:

$$dx/dt = f(x,V),$$
$$I(x,V) = Y \cdot V, \tag{1}$$

where x is the state vector representing dynamic variables, f is a nonlinear function characterizing the system dynamics, V represents nodal voltages, Y is the nodal admittance matrix, I is the injected current vector.

The integration of these equations provides an accurate depiction of both the transient and steady-state responses of the power systems. These equations encapsulate the interaction between the electrical network and dynamic system elements, such as transformers, synchronous machines, and power electronic converters. Specifically, the differential equations model the time-dependent dynamics of the system components, while the algebraic equations enforce network constraints and capture the interconnection of elements through nodal admittance and current injections.

The differential equations represent the system elements' dynamics (transformers, induction motors, synchronous machines and their controllers, power electronics, etc.) while the algebraic equations represent the network equations and the connection of the external elements to the network.

The solution methodology for addressing differential and algebraic equations can be categorized into two main strategies: alternating and simultaneous approaches [12]. The alternating scheme operates by separately solving the systems of ordinary differential equations (ODEs) and algebraic equations. It relies on foundational theories, including ODE solution methodologies, algebraic equation-solving techniques, and state-space modeling principles [11, 12]. Conversely, the simultaneous scheme directly addresses the coupled system of differential-algebraic equations (DAEs), leveraging the theoretical frameworks of DAE solution methods and the generalized state-space modeling approach [13].

The methodologies for solving the given equations are classified into two strategies: alternating schemes and simultaneous schemes. Alternating schemes solve differential and algebraic equations independently, leveraging traditional theories such as state-space modeling and iterative algebraic techniques [14]. On the other hand, simultaneous schemes tackle the coupled system of DAEs directly, utilizing modern approaches like the generalized state-space model and advanced numerical solvers for DAEs [15, 16].

Recent advancements in computational techniques, including adaptive solvers and hybrid symbolic-numeric methods, have significantly improved the efficiency and accuracy of these solution schemes. These innovations are pivotal for analyzing increasingly complex systems, such as hybrid renewable energy networks and large-scale power grids [15, 17].

GSSM description. GSSM comprises a system of differential and/or algebraic equations, which for linear time-invariant dynamic systems are [18-20]:

$$\begin{aligned} E\,\dot{x}(t) &= A\,x(t) + B\,u(t), \\ y(t) &= C\,x(t) + D\,u(t), \end{aligned} \tag{2}$$

where x is the state vector, u is the input vector, y is the output vector, E, A, B, C, D are constant matrices of appropriate dimensions.

If E is non-singular, the system (2) can be represented as:

$$\begin{aligned} \dot{x}(t) &= E^{-1}A\,x(t) + E^{-1}B\,u(t), \\ y(t) &= C\,x(t) + D\,u(t), \end{aligned} \tag{3}$$

which is the widely used state-space representation.

Similarly, if $E = I$ (where I is the identity matrix) the system (2) is the regular state-space model:

$$\begin{aligned} \dot{x}(t) &= A\,x(t) + B\,u(t), \\ y(t) &= C\,x(t) + D\,u(t). \end{aligned} \tag{4}$$

Thus, the GSSM (2) describes a broader general class of dynamic systems than the regular GSSM (4). Normally, power systems are nonlinear time-varying circuits. According to [21] GSSM method is the most appropriate for computer simulation of nonlinear time-varying systems. The dynamic response of nonlinear time-varying systems is generally represented by a set of differential equations that model the time-dependent changes in the systems components, such as capacitors and inductors, alongside algebraic equations that capture the relationships imposed by network constraints, such as Kirchhoff's laws [22]:

$$F(\dot{x}, x, u, t) = 0,$$
$$G(y, x, u, t) = 0, \tag{5}$$

where x is the state vector, u is the input vector, y is the output vector, F, G are nonlinear vector functions.

However, the proposed approach is complex due to the fact that model (5) is overly intricate, making it impractical for most analysis, synthesis, or optimization tasks. The conventional state-space model of nonlinear time-varying systems is represented by the following equations' set:

$$\dot{x} = F(x, u, t),$$
$$y = G(x, t). \tag{6}$$

The advantage of this approach lies in the well-established theory of mathematical and computational modeling of electrical circuits using the state-space method. However, the challenge related to model (6) is that it requires separate, element-by-element calculations for the nonlinear function increments of GG, which complicates the process.

The canonical form of the generalized state-space model for nonlinear time-varying circuits is outlined below [23]:

$$E\,\dot{x} = A(x,t) + Bu,$$
$$y = C\,x, \tag{7}$$

where B, C, E are constant matrixes. This form is utilized for power systems, which are typically challenging to simulate due to the presence of algebraic equations and complex nonlinearities. The system of nonlinear differential-algebraic equations (DAEs) must be solved. Unlike explicit ordinary differential equations, integrating DAEs can present significant challenges [24]. Constraints define a manifold within which the solutions must reside, and the initial values must be selected to ensure they satisfy these constraints. Additionally, the numerical solution must remain sufficiently close to the manifold to avoid significant deviation, ensuring that the system's dynamics remains valid throughout the integration process.

DAEs essentials. DAEs are a class of equations that combine differential equations with algebraic constraints, making them essential for modeling systems with dynamic and static components. Unlike ordinary differential equations (ODEs), DAEs incorporate relationships that may not involve derivatives, leading to structural complexities in analysis and solution methods]25]. DAEs are implicitly defined ODEs that are inherently singular in nature [26]:

$$F(x', x, t) = 0, \tag{8}$$

where the partial Jacobian $\partial F / \partial x'$ identically singular for all values of its arguments. Depending on the area of application, DAEs are also called implicit, descriptor or singular [25, 26]. If $\partial F / \partial x'$ were nonsingular, equation (8) could, at least theoretically, be solved explicitly for x', at least theoretically, resulting in a standard ODE. In essence, DAEs can be viewed as a form of ODEs; however, they are characterized by the fact that they cannot be explicitly solved for x' [27]. The presence of algebraic constraints in F differentiates DAEs from ODEs. A fundamental characteristic of DAEs is their differentiation index, which quantifies the number of times the equations need to be differentiated to reformulate them as a system of ODEs. Solving higher-index DAEs is particularly challenging due to numerical instabilities and the risk of inconsistent initial conditions, making their analysis and computation more complex [5]. DAEs) are extensively utilized in engineering and scientific domains to model systems with inherent physical constraints or interdependencies, particularly in power systems to model electrical circuits and power grids governed by Kirchhoff's laws. These equations account for both dynamics, such as inductive and capacitive interactions, and algebraic constraints, such as current and voltage conservation, enabling accurate simulation of complex networks. Such models are essential for stability analysis, fault detection, and optimization of modern energy systems [28].

Analytical consideration. To illustrate the unique characteristics of DAEs, consider the following simple example [29]:

$$x_2' = x_1, \tag{9}$$

$$x_2 = t + \alpha(t) x_3, \tag{10}$$

$$x_3' = x_3 + 1, \tag{11}$$

where $\alpha(t)$ is a nonzero coefficient.

The solution of the given equations; system is represented as follows:

$$x = \begin{bmatrix} 1 + \alpha'(t)(-1 + c e^t) + c\alpha(t) e^t \\ t + \alpha(t)(-1 + c e^t) \\ -1 + c e^t \end{bmatrix}, \tag{12}$$

where c is an arbitrary constant.

The following case highlights several key distinctions between DAEs and ODEs [29, 30]:

- solution x of (12) can depend on derivatives of the defining equation F (notice a term $\alpha'(t)$ that appears in the solution);
- not all initial conditions result in smooth solutions, the initial conditions that provide smooth solutions are referred to as «consistent initial conditions»;

- in addition, some hidden constraints are expected, solution of (12) satisfy not only the constraint (10) but also the following constraint:

$$\alpha'(t)\,x_3 + \alpha(t)(1 + x_3) - x_1 = 0\;;$$

- the best possible outcome is that the solution lies on a smooth manifold, known as the «solution manifold». This manifold is parameterized by *t*, *c*, as shown in the example above.

As previously mentioned, DAEs are singular equations, and their singularity is quantified by a nonnegative integer known as the «index». The concept of the index enables the classification of DAEs based on their solution dynamics. Several definitions of the index exist, including global index, geometrical index, perturbation index, differential index, and tractability index, which are consistent for simple systems (e.g., linear DAEs with specific assumptions). However, these indices can vary for more complex systems, such as nonlinear DAEs. Notably, ODEs have an index of zero, as they represent the simplest form of differential equations.

One of the index definitions is the number of times a DAE must be differentiated in order to transform it into an explicit ODE involving all state variables. This is referred to as the «differentiation index». It quantifies the complexity of the system by measuring the number of differentiations required to reduce a DAE to a simpler form comparable to an ODE [31].

Numerical consideration. DAEs of higher indices (greater than 1) are ill-posed, meaning that small perturbations in the initial conditions can lead to large, unpredictable changes in the solution [25, 27, 29]. As a result, classical numerical methods are not universally applicable to all DAEs. index-1 DAEs, however, are the most widely studied, and methods like backward differentiation formulas (BDF), implicit Runge-Kutta (IRK), and various extrapolation techniques are effective for solving them. Common solvers for index-1 DAEs include DASSL, DASPK, LSODI, RADAU5, and CHORAL. For DAEs with indices 2 or higher, classical methods typically only work for specific structured systems, such as Hessenberg systems. There is no general solver for index-2 DAEs, though specialized methods exist for certain applications, such as constrained mechanics and electrical circuits.

Computer experiment. In the computer experiment, a Power Supply Control Device (PSCD) was simulated. This type of devices are used to initiate inductive motors in low power supply networks. Figure 1 illustrates the electrical schematic, while Figure 2 shows the equivalent circuit diagram for a single phase of the PSCD.

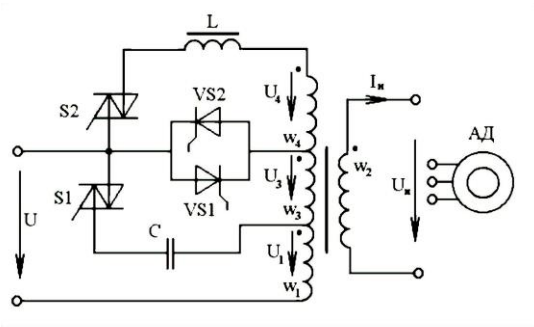

Fig. 1. *Electrical schematic for one phase of PSCD*

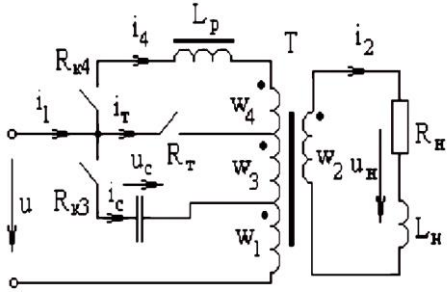

Fig. 2. *Equivalent diagram of one phase of PSCD*

PSCD dynamics can be simulated by a set of nonlinear ODEs and a set of nonlinear algebraic equation as follows:

$$\begin{cases} \Psi_1' = u - R_1 i_1 - u_C - R_{к3}(i_1 - i_3), \\ \Psi_2' = -(R_н + R_2) i_2, \\ \Psi_3' = u_C - R_3 i_3 - R_m(i_3 - i_4) + R_{к3}(i_1 - i_3), \\ \Psi_4' = R_m(i_3 - i_4) - (R_{к3} + R_4 + R_p) i_4, \\ u_C' = \dfrac{1}{C}(i_1 - i_3), \end{cases} \tag{13}$$

$$\begin{cases} (L_1 + L_{\sigma 1}) i_1 + M_{12} i_2 + M_{13} i_3 + M_{14} i_4 = \Psi_1, \\ M_{21} i_1 + (L_2 + L_{\sigma 2} + L_н) i_2 + M_{23} i_3 + M_{24} i_4 = \Psi_2, \\ M_{31} i_1 + M_{32} i_2 + (L_3 + L_{\sigma 3}) i_3 + M_{34} i_4 = \Psi_3, \\ M_{41} i_1 + M_{42} i_2 + M_{43} i_3 + (L_4 + L_{\sigma 4} + L_p) i_4 = \Psi_4, \\ u_н = -(R_н + R_2) i_2, \end{cases} \tag{14}$$

where $\Psi_1 \dots \Psi_4$ are the PSCD winding fluxes; $i_1 \dots i_4$ are the winding currents; u, $u_н$, u_C are the input, output and capacitor voltages correspondingly; $R_1 \dots R_4$, $R_н$ are active resistances of transformer windings and load; $R_{к3}$, $R_{к4}$, R_m are correspondingly resistances of controlled switches; C is the capacitor capacity; $L_н$ is the load inductance; $L_j = \dfrac{w_j^2}{R_\mu}$, $j = \overline{1,4}$ are the of transformer windings' self-inductances; $L_{\sigma 1} \dots L_{\sigma 4}$ are the transformer windings' leakage inductances; $M_{ij} = \dfrac{w_i \cdot w_j}{R_\mu}$, $i, j = \overline{1,4}$ are the transformer windings' common inductances; $w_1 \dots w_4$ are the winds' number; R_μ is the magnetizing resistance, that is non-linear function of the transformer windings ampere turns' sum.

Through appropriate transformations of equations (13) and (14), we can derive the generalized state-space model for the PSCD as follows.

$$\begin{cases} E\,\dot{x} = Ax + Bu, \\ y = C_1\,x, \end{cases} \tag{15}$$

where

$$x = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ u_C \end{bmatrix}, \; y = u_н\,,$$

$$E = \begin{bmatrix} L_1 + L_{\sigma 1} & M_{12} & M_{13} & M_{14} & 0 \\ M_{21} & L_2 + L_{\sigma 2} + L_н & M_{23} & M_{24} & 0 \\ M_{31} & M_{32} & L_3 + L_{\sigma 3} & M_{34} & 0 \\ M_{41} & M_{42} & M_{43} & L_4 + L_{\sigma 4} + L_p & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A = \begin{bmatrix} -(R_1 + R_{к3}) & 0 & R_{к3} & 0 & -1 \\ 0 & -(R_н + R_2) & 0 & 0 & 0 \\ R_{к3} & 0 & -(R_3 + R_m - R_{к3}) & -R_m & 1 \\ 0 & 0 & R_m & -(R_m + R_{к3} + R_4 + R_p) & 0 \\ \dfrac{1}{C} & 0 & -\dfrac{1}{C} & 0 & 0 \end{bmatrix},$$

$$B=\begin{bmatrix}1&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix},\ C_1=\begin{bmatrix}0&0&0&0&0\\0&-(R_н+R_2)&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}.$$

The following PSCD dynamics was simulated during the computer experiment:

- capacitor branch is connected and inductance branch is switching-off: $R_{к3} = 0$, $R_{к4} = \infty$, R_m is regulated;
- inductance branch is connected and the capacitor branch is switching-off: $R_{к3} = \infty$, $R_{к4} = 0$, R_m is regulated;
- both branches are switching-on in the resonant mode: $R_{к3} = 0$, $R_{к4} = 0$, $R_m = \infty$.

Figure 3 illustrates the PSCD dynamics for the following case: $R_{к3} = 0$, $R_{к4} = \infty$ and the gate turn-off thyristors R_m are pulse-position modulated. While the Fig. 4 highlights the PSCD dynamics of MDPR for the following case: $R_{к3} = 0$, $R_{к4} = \infty$ and the gate turn-off thyristors R_m are pulse-width modulated.

The PSCD parameters were as follows:

$w_1 = 1300$; $w_2 = 52$; $w_3 = w_4 = 130$; $C = 79{\cdot}10^{-6}$ F; $R_1 = 3.397\ \Omega$;
$R_2 = 0.0492\ \Omega$; $R_3 = R_4 = 0.3394\ \Omega$; $R_н = 0.43\ \Omega$; $R_p = 0.5\ \Omega$; $R_\mu = 14300\ \Omega$;
$L_{\sigma 1} = 0.028$ H; $L_{\sigma 2} = 3.8{\cdot}10^{-5}$ H; $L_{\sigma 3} = L_{\sigma 4} = 0.0029$ H; $L_н = 0.0013$ H.

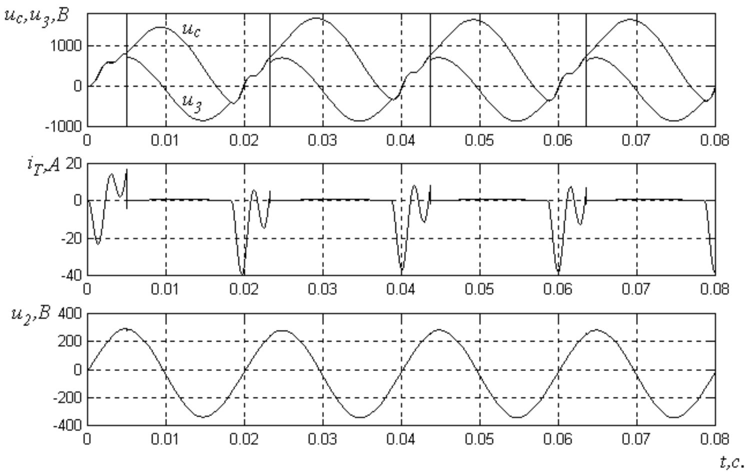

Fig. 3. *PSCD dynamics: pulse-position modulated control*

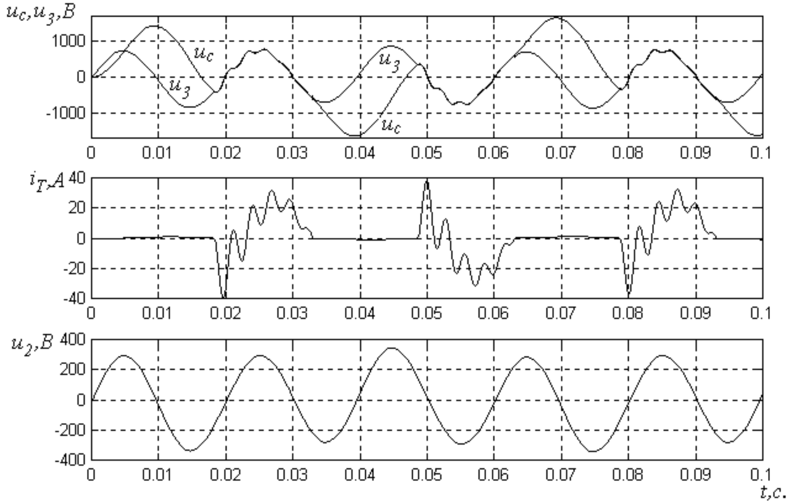

Fig. 4. *PSCD dynamics: pulse-width modulated control*

The given computer experiments showed that the application of the GSSM for power systems' dynamics simulation significantly streamlines the modeling process. It achieves this by reducing the number of required modeling equations (e.g., from 10 to 6 for the given case) and employing a unified algorithm to solve the resulting set of differential-algebraic equations (DAEs). In contrast, traditional methods necessitate two separate algorithms: one for solving ordinary differential equations (ODEs) and another for algebraic equations. This consolidation simplifies the computational workflow, enhances efficiency, and reduces the complexity of dynamic simulations.

Conclusions. This study focused on the simulation of power systems' dynamics using the GSSM, assessing its potential as a tool for modern power system modeling. The GSSM approach demonstrated several key benefits over traditional methods, including almost two-times reduction in the number of modeling equations and the ability to use a single algorithm for solving differential-algebraic equations (DAEs), while conventional methods require separate algorithms for ordinary differential equations (ODEs) and algebraic equations, complicating the computational process. The given results can confirm the feasibility of the GSSM approach for power systems' dynamics simulation, and assess GSSM as a promising approach for addressing the challenges of modern power systems characterized by hybrid dynamics, nonlinear interactions, and complex switching operations, particularly:

- within simplifying the modeling process and reducing computational complexity by consolidating continuous and discrete dynamics into a unified framework;
- within employing a unified algorithm for DAE solving, which eliminates the need for disparate computational strategies, making the modeling process more efficient and scalable;
- within identifying critical challenges that need to be addressed to enhance the applicability of GSSM.

At the same time, further research is required to address existing limitations and enhance its practicality. This includes the development of efficient and stable numerical algorithms tailored for solving differential-algebraic equations (DAEs) that arise in such simulations. Additionally, the creation of intelligent software routines capable of automating the DAE-solving process is essential to enhance usability and adaptability. Furthermore, advancing methods for the automated generation of dynamic state equations within the GSSM framework is crucial for reducing manual intervention and improving scalability. Addressing these issues can significantly enhance the efficiency and applicability of the GSSM approach in complex power systems' simulations.

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ПРО ЗАСТОСУВАННЯ МЕТОДУ GSSM У МОДЕЛЮВАННІ ДИНАМІКИ ЕНЕРГЕТИЧНИХ СИСТЕМ

Розглянуто широкий спектр аспектів, пов'язаних з доцільністю, принципами та реалізацією узагальненої моделі простору станів (GSSM) як математичної техніки моделювання для моделювання енергетичних систем. У статті оглядається використання GSSM для вирішення проблем адекватного моделювання динаміки сучасних енергетичних систем, які характеризуються їх гібридною природою, складними режимами перемикання та нелінійними взаємодіями. Аналізується універсальність GSSM у порівнянні з традиційними підходами, зокрема його здатність інтегрувати як безперервну, так і дискретну динаміку систем у єдину покращену структуру. Також обговорюється здатність моделі захоплювати як лінійні, так і нелінійні режими, її сумісність із сучасними обчислювальними інструментами та застосування в різних енергетичних системах. Окремо розглядаються основи, аналітичні та числові аспекти диференціально-алгебраїчних рівнянь (DAE). Ефективність GSSM демонструється через комп'ютерний експеримент з моделювання динаміки пристрою для регулювання енергопостачання. Експерименти з комп'ютерного моделювання підкреслюють переваги GSSM порівняно з традиційними методами з точки зору точності, обчислювальної ефективності та масштабованості. Водночас визначаються області, де необхідні подальші дослідження та удосконалення.

Ключові слова: *енергетичні системи, узагальнена модель простору станів, диференціально-алгебраїчні рівняння, математичне моделювання, динаміка енергетичної системи.*

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