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MODELING NONLINEAR ENERGY TRANSMISSION CHAINS VIA VOLTERRA SERIES: CONVERGENCE CRITERIA AND STABILITY BOUNDS UNDER PERIODIC EXCITATION

This paper derives sufficient conditions for the convergence of Volterra series representing solutions to a class of nonlinear integral equations that model energy objects' dynamic networks with periodic input signals. By formulating the system's response through a nonlinear integral equation, we establish rigorous criteria for the absolute convergence of the Volterra series expansion. Specifically, we analyze energy objects' networks containing ideal bandpass filters excited by trigonometric polynomial inputs, a configuration common in simplified analyses of physically realizable systems. For energy systems' resistive nonlinear chains, we demonstrate that the Volterra series reduces to a power series and provide explicit estimates of its convergence radius (Theoretical statement 3). Additionally, Theoretical statements 1 and 2 present generalized convergence criteria based on the minimization of a functional over a constrained spatial domain, extending prior results for NARX-type systems. The results can contribute to bridging theoretical analysis with engineering applications, offering practical tools for designing nonlinear energy systems' chains with predictable dynamics, such as those found in power electronics and signal processing systems.

Key words: *Volterra series convergence, nonlinear integral equations, dynamic networks, periodic input signals, resistive nonlinear chains, convergence radius estimation, power series approximation, nonlinear energy systems.*

Introduction. The Volterra series has long served as a foundational tool in the analysis and modeling of nonlinear systems, particularly in contexts where memory effects and weak nonlinearities are present. Unlike the Taylor series, which is limited to memoryless systems, the Volterra series captures dynamic behavior through multidimensional convolution integrals, making it indispensable for describing nonlinear chains of energy objects such as resistive or reactive networks. However, the practical utility of the Volterra series hinges critically on the convergence of the expansion, especially when modeling complex nonlinear chains subjected to periodic or broadband excitations.

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Recent advances have highlighted the limitations of classical convergence criteria, particularly for systems modeled by NARX (Nonlinear Auto-Regressive with Exogenous Input) structures. Zhu and Lang (2020) [1] introduced a Generalized Output Bound Characteristic Function (GOBCF) to rigorously assess convergence, offering a more robust alternative to earlier methods. Further refinements in frequency-domain approaches, as detailed in Zhu's 2021 monograph [2], have expanded the applicability of Volterra series to higher-order nonlinearities and general input signals. Additionally, Jing and Xiao (2017) [3] derived analytical bounds for convergence in a class of nonlinear systems, providing key insights into the radius of convergence under various excitations. The works [1-3] explore both theoretical and practical aspects of convergence, including the use of the Generalized Output Bound Characteristic Function (GOBCF), frequency-domain representations, and parametric bounds for nonlinear systems under various input conditions.

Despite these advancements, the increasing complexity of modern energy systems-from smart grids to nonlinear transmission chains-demands more precise convergence guarantees. This paper addresses this need by establishing sufficient conditions for the convergence of Volterra series in a specific class of nonlinear dynamic chains. By modeling these systems through nonlinear integral equations, we derive explicit criteria for convergence and estimate the convergence radius for resistive chains under periodic inputs. The results not only can extend the theoretical framework of nonlinear system analysis but also can provide practical insights for engineers designing energy systems with predictable dynamic responses.

One of the methods for analyzing nonlinear systems is the method of Volterra series [4-6]. This method is usually used to calculate the steady state of systems in order to determine the mutual influence of nonlinear distortions. Literature sources also provide examples of how this method is used to calculate oscillations in autonomous systems [7]. An extremely important task associated with the use of Volterra series is convergence and determination of truncation error conditions of local convergence [8-10], as well as nonlocal convergence of Volterra series [1, 11-13] for a certain class of systems described by equations of state and integral equations. In this work, an attempt is made to establish sufficient conditions and radius of convergence of Volterra series for a certain class of networks.

Dynamic network analysis: computational methods for nonlinear systems with periodic excitations. A wide class of nonlinear networks containing a single nonlinear element (Fig. 1),

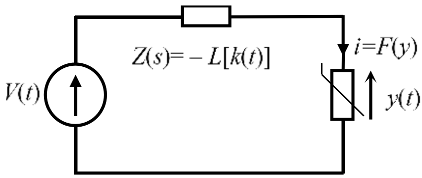

Fig. 1. *Network with a nonlinear resistor*

can be described by the following integral equation

$$y(t) = \int_0^t k(t-\tau)F\left[y(\tau)\right]d\tau + v(t) \quad t \in [0,\infty), \tag{1}$$

where $k(t)$ is the impulse response of the linear part of the network for which $\int_{-\infty}^{\infty} |k(t)|dt < \infty$, $v(t)$ is the input signal, $y(t)$ is the output signal, $F : |R \to| R$ an integer function characterizing a nonlinear element with the property. $F(0) = F^{'}(0) = 0$. Let us assume that the input $v(t)$ is a periodic signal (not necessarily valid) with a period $T = 2\pi/\Omega$ described by a trigonometric polynomial

$$v(t) = \sum_{n=-N}^{N} v_n e^{jn\Omega t}, \quad v_n \in C \,. \tag{2}$$

Let us analyze the established reaction $y(t)$. Consider an equation describing a steady state

$$y(t) = \int_0^{\infty} k(\tau)F\left[y(t-\tau)\right]d\tau + v(t), \quad t \in (-\infty,\infty). \tag{3}$$

The solution of equation (3) can be presented in the following form

$$y(t) = \sum_n y_n e^{jn\Omega t} \,. \tag{4}$$

To analyze equation (3), we apply the method of Volterra series, based on the representation of the solution $y(t)$ in the form of a homogeneous mapping V_n

$$y(t) = \sum_{n=1}^{\infty} y_n v(t) = \sum_{n=1}^{\infty} \int_0^{\infty} \cdots \int_0^{\infty} g_n(t-\tau_1,\cdots,t-\tau_n) \prod_{i=1}^{n} v(\tau_i)\, d\tau_i \,, \tag{5}$$

where $g_n(\tau_1,\ldots,\tau_n)$ is the nucleus of Volterra of the nth order.

Substituting (2) in the row (5), we get the steady state $y(t)$ in the following form

$$y(t)=\sum_{n=1}^{\infty}\sum_{i_1\ldots,i_n=-N}^{N}G_n\left(p_{i_1},\ldots,p_{i_n}\right)v_{i_1}\ldots v_{i_n}e^{j\left(p_{i_1}+\ldots+p_{i_n}\right)t}\ , \tag{6}$$

where $G_n\left(s_1,\ldots,s_n\right)$ is the nth order transfer function, $p_{i_k}=\Omega_{ik}$. Formula (6) shows that each element of series (4) has the form of a power series with $2N+1$ complex variables

$$y_n=\sum_{\alpha_1,\ldots,\alpha_{2N+1=0}}^{\infty}a^{(n)}_{\alpha_1,\ldots,\alpha_{2N+1}}\quad v_{-N}^{\alpha_1},\ldots,v_N^{\alpha_{2N+1}},\ n=0,+1,\ldots;\ a_\alpha^{(n)}\in C\ . \tag{7}$$

In addition, suppose that there $y(t)$ are no harmonic components in the reaction. This means that we must try to find a solution to the following equation

$$y(t)=\int_{-\infty}^{\infty}k(\tau)F\left[y(t-\tau)\right]d\tau+v(t),\quad t\in(-\infty,\infty)\ . \tag{8}$$

The conditions for uniform convergence with respect to the time of t the series $\sum_n y^{(n)}(t)$ to the solution $y(t)$ of equation (8) can be represented as follows

$$\underline{y}-\underline{U}\,\underline{y}=\underline{v}\ , \tag{9}$$

where

$$\underline{U}\,\underline{y}=\left[K(-jN\Omega)u_{-N}\left(y_{-N},\cdots,y_N\right),\cdots,K(jN\Omega)u_N\left(y_{-N},\cdots,y_N\right)\right]^T,$$

and

$$u_n\left(y_{-N},\cdots,y_N\right)=\frac{1}{T}\int_0^T F\left(\sum_{k=-N}^{N}y_k e^{jk\Omega t}\right)e^{-jk\Omega t}dt,\quad n=0,\pm1,\cdots,+N\ .$$

Thus, the method of Volterra series consists in finding a solution $y=\left[y_{-N},\ldots,y_N\right]^T$ to a system with $2N+1$ nonlinear equations (9) in a complex Euclidean space in the C^{2N+1} form of a power series. Let us determine the radius of convergence of the power series.

Definition 1. The radius of convergence is the maximum radius of a ball centered at zero in which the series (7) is absolutely converging and is written as

$$1/\rho_n=\sup_{\sum_{\eta=-N}^{N}\left|v_n\right|^2=1}\left[\lim_{k\longrightarrow\infty}\sup\sqrt[k]{\sum_{\alpha_{1+\ldots+}\alpha_{2N+1}=k}\left|a^{(n)}_{\alpha_{1,\cdots,}\alpha_{2N+1}}v_{-N}^{\alpha_1}\cdots v_N^{\alpha_{2N+1}}\right|}\right]. \tag{10}$$

Particularly, we're looking for a the radius of a sphere ρ in which all rows (7) are absolutely converging for $n=0,\pm1,\ldots,+N$, which means $\rho=\min_n\rho_n$.

The power series in the convergence ball are uniformly converging, hence the Volterra series (5) is uniformly converging in the time domain t.

Convergence criteria for series expansions in nonlinear systems. It follows from equation (9) that $(\underline{1}-\underline{U})\underline{y}=\underline{v}$. The mapping $(\underline{1}-\underline{U})$ is holomorphic for everyone $\underline{y}\in C^{2N+1}$, since F We require that the inverse mapping to $\underline{G}=(\underline{1}-\underline{U})^{-1}$ also be holomorphic on a specified solution set. A mapping possessing this property is called a biholomorphic mapping. It is known that a mapping is biholomorphic if and only if it is both injective and holomorphic.

To determine the domain in which the operation $(\underline{1}-\underline{U}):D_y\to D_v$ is injective, let's apply the Banach compression theoretical statement. The Fréchet derivative of the mapping $\underline{U}$ in C^{2N+1} is a Jacobian matrix in the form

$$\underline{U}'(\underline{y})=\begin{bmatrix} K_{-N}\partial u_{-N}(\underline{y})/\partial y_{-N}\cdots K_{-N}\partial u_{-N}(\underline{y})/\partial y_{N} \\ K_{N}\partial u_{N}(\underline{y})/\partial y_{-N}\cdots K_{N}\partial u_{N}(\underline{y})/\partial y_{N} \end{bmatrix}, \qquad (11)$$

$$K_n=K(jn\Omega).$$

It follows from Banach's theoretical statement that a set of elements satisfying the inequality $\left\|\underline{U}'(\underline{y})\right\|<1$ or

$$\max_{1\le n\le 2N+1}\lambda_n(\underline{y})<1\ , \qquad (12)$$

where $\lambda_1,...,\lambda_{2N+1}$ are the characteristic numbers of the matrix $\left(\underline{\bar{U}}'\right)^T\underline{U}'$, forms the domain D_y in which the mapping $(\underline{1}-\underline{U})$ is injective. Therefore, the mapping $(\underline{1}-\underline{U})$ of the domain D_y in D_v is injective and holomorphic. The radius ρ_a centered at zero included in D_v, expressing sufficient conditions for the convergence of the Volterra series, is equal to the smallest distance of the point to the $v=0$ boundary of the Γ_v domain D_v, which means

$$\rho_a=\min_{\underline{y}\in\Gamma_y}\left\|\underline{y}-\underline{U}_{\underline{y}}\right\|=\min_{\underline{v}\in\Gamma_v}\left\|\underline{v}\right\|. \qquad (13)$$

The above propositions can be summarized in the form of the following theoretical statement:

Theoretical statement 1. If the following assumptions are met:

1. $K(j\omega)=0$ where $|\omega|>N\Omega$;

2. F – integer function and $F(0)=F^{'}(0)=0$, then for signals $v(t)$ satisfying the inequality $\sum_{n=-N}^{N}|v_n|^2<\rho^2 a$, the Volterra series is uniformly converging with respect to time to t the solution of equation (8) taking into account the expression (13).

A set in which the mapping $(\underline{1}-\underline{U})$ is biholomorphic, can be determined by a different method using Rouché's theorem [7], which requires some additional constraints on the functions u_n.

Consider the following theoretical statement that defines sufficient conditions for the convergence of Volterra series.

Theoretical statement 2. Let assumptions 1 and 2 of theoretical statement 1 hold and the functions u_n satisfy the conditions:

1. $$u_n\left(y_{-N},\cdots,y_N\right)\Big|_{y_{n=0}}=0 \text{ where } n=0,\pm1,\cdots,+N, \qquad (14)$$

2. $y_n=K_n u_n\left(y_{-N},\ldots,y_N\right)\neq 0$ where $y_n\neq 0$ then
$$\underline{y}\in P\left(0,R_{-N},\ldots,R_N\right). \qquad (15)$$

Then for the input signal $v(t)$, satisfying the inequality

$$|v_n|<M_n\left(r_{-N},\ldots,r_N\right)=\inf_{|y_{-N}|=r_{-N}\ldots|y_N|=r_N}\left|y_n-K_n u_n\left(y_{-N},\ldots,y_N\right)\right| \qquad (16)$$
$$n=0,\pm1,\ldots,+N.$$

The Volterra series is uniformly converging to the t solution of equation (8), where $0<r_n<R_n$.

Assumption (14) significantly limits the class of functions u_n and F, for which we can use Theoretical statement 2, whereas inequality (15) will always be satisfied in the class of systems considered.

Instead of using the inequality (16) that defines the amplitude range of the individual harmonics of the input signal $v(t)$, we can use the formula

$$\sqrt{\sum_{n=-N}^{N}|v_n|^2}<\rho\left(r_{-N},\ldots,r_N\right)=\min_{-N\le n\le N}M_n\left(r_{-N},\ldots,r_N\right),$$

determining the effective value $v(t)$ of. Primary interest lies in the maximal achievable signal value ρ_b, given by the expression

$$\rho_b=\max_{\substack{0<r<R_{-N}\\ \vdots \\ 0<r<R_N}}\rho\left(r_{-N},\ldots,r_N\right).$$

Nonlinear resistive networks: Volterra series analysis and convergence properties. Using Theoretical statement 2, we can determine the necessary and sufficient conditions for the convergence of Volterra series for specific types of networks and signals. Consider the resistive network (Fig. 2) described by equation (1), which for $k(t)=k_0\delta(t)$ takes the form of

$$y=k_0F\left(y\right)+v. \tag{17}$$

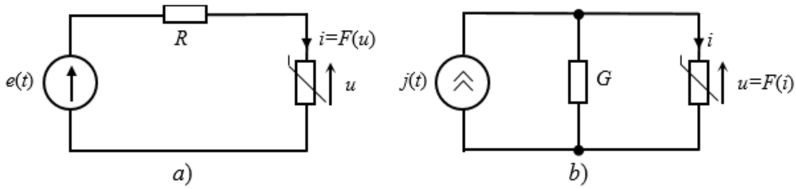

Fig. 2. *Nonlinear resistive network*

Let us solve equation (17) in the form of the Volterra series (5), which in our case is a power series $y=\sum_n a_n v^n$ converging for $|v|<\rho_c$.

Within the convergence domain, the power series converges uniformly. Then for the signals $v(t)$, continuous and limited, satisfying the condition of $\sup_t |v(t)|<\rho_c$ row $\sum_n a_n\left[v(t)\right]^n$ is uniformly converging in t to a continuous and limited function $y(t)$. Radius of convergence ρ_c of such a series is defined by theoretical statement 3.

Theoretical statement 3. Let the nonlinear element be described by the analytic function $F(y)=\sum_{n=2}^{\infty} c_n y^n$ for $|y|<r_a$. The radius of convergence of the Volterra series, which is the solution of equation (17), is given by the following expression

$$\rho_c=\sup_{0<r<R}\ \inf_{0\le\varphi\le 2\pi}\left|re^{j\varphi}-k_0F\left(re^{j\varphi}\right)\right|, \tag{18}$$

where is $R<r_a$ the radius of the largest domain in which $y-k_0F(y)\ne 0$ for $y\ne 0$.

Practical case. The practical application of the convergence criterion defined by theoretical statements 1, 2 and 3 can be illustrated by the following example. Consider the chain defined by the equation

$$y\left(t\right)=\int_{-\infty}^{\infty} k\left(\tau\right)\left[y\left(t-\tau\right)\right]^3 d\tau+A\cos\Omega t. \tag{19}$$

The frequency response of the linear part of this network is shown in Fig. 3.

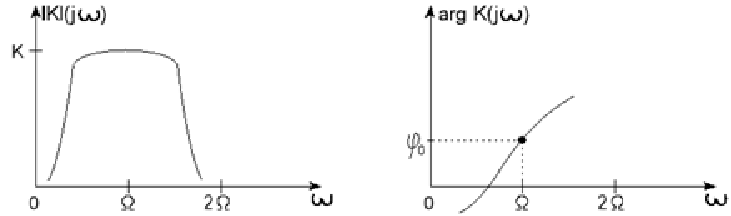

Fig. 3. *Frequency response of the linear part*

The members of the series determined on the basis of the recurrent formula are

$$y(t) = A\cos\Omega t + \frac{3}{4}KA^3\cos(\Omega t + \varphi_0) + \frac{9}{8}K^2A^5\left[\cos(\Omega t + 2\varphi_0) + \frac{1}{2}\cos\Omega t\right] +$$

$$+\frac{27}{16}K^3A^7\left[\cos(\Omega t + 3\varphi_0) + \frac{3}{4}\cos(\Omega t + \varphi_0) + \frac{1}{2}\cos(\Omega t - \varphi_0)\right] + \ldots$$

Equation (9) becomes

$$\begin{bmatrix} y_{-1} \\ y_1 \end{bmatrix} - \begin{bmatrix} 3K_{-1}y_{-1}^2 y_1 \\ 3K_1 y_{-1} y_1^2 \end{bmatrix} = \begin{bmatrix} v_{-1} \\ v_1 \end{bmatrix}. \qquad (20)$$

It follows from theoretical statement 1 that

$$A^2 \big/ 2 < \rho_a^2 = \frac{2}{\sqrt{90}K}\left[1 + \frac{3}{\sqrt{90}}\left(\frac{3}{\sqrt{90}} - 2\cos\varphi_0\right)\right].$$

The analyzed chain satisfies the assumptions of theoretical statement 2. Let's also assume that $v_{-1} = \overline{v}_1$. From theoretical statement 2 we get

$$A \big/ 2 < \max_{0<r<1/\sqrt{3K}} r\left(1 - 3Kr^2\right) = \frac{2}{9\sqrt{K}}. \qquad (21)$$

In a particular case, when $\varphi_0 = 0$, the system of two equations (20) can be reduced to one equation for the amplitude

$$|y_i| = 3K|y_i|^3 + |v_i| \qquad (22)$$

and $\arg y_i = \arg v_i$.

Using equation (22), theoretical statement 3 (or another method), it is possible to prove that the radius of convergence of the series $|y_i| = \sum_n a_n |v_i|^n$ is $|v_i| < \rho_c = 2/(9\sqrt{K})$. Thus, using theoretical statement 2 in the case $\varphi_0 = 0$, we obtain the quantity (21), which determines the radius of convergence of the Volterra series. The dependence of the obtained sufficient conditions for the convergence of the Volterra series, which is the solution of equation (19) in the phase function φ_0, is shown in Fig. 4, where $T1 < T2$ and T3 – are the corresponding theoretical statements.

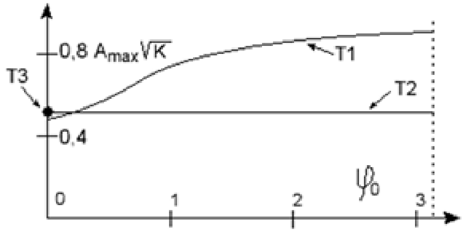

Fig. 4. *Solution of equation (19) in the phase function*

Conclusion. This paper has presented a rigorous method for estimating the radius of convergence of the Volterra series as a solution to a nonlinear integral equation, with a focus on networks containing an ideal bandpass filter under periodic excitation in the form of a trigonometric polynomial. Such a formulation is particularly relevant for the simplified analysis of physically realizable nonlinear systems, where the periodic response is often approximated by a finite number of harmonic components.

The key contributions of this work are as follows:

1. Convergence criteria for nonlinear integral equations (Theoretical statements 1 and 2):
   - sufficient conditions for the convergence of the Volterra series by deriving a criterion based on the minimization of a specific functional over a defined spatial domain are established;
   - these results can extend classical convergence analysis by incorporating periodic input constraints, ensuring applicability to real-world energy engineering problems, such as nonlinear filter design and signal processing.
2. Explicit radius of convergence for resistive chains (Theoretical statement 3):
   - for the class of resistive nonlinear chains described by equation (17), we demonstrated that the Volterra series reduces to a power series with a well-defined radius of convergence;
   - this provides a computationally tractable framework for engineers to assess the validity of Volterra series approximations in practical network analysis.

Broader implications and future work. The derived convergence conditions are particularly valuable in nonlinear network theory, where ensuring the validity of functional expansions is crucial for accurate modeling. Future research could explore generalizations to time-varying systems or stochastic excitations, as well as numerical implementations for automated convergence verification in network simulation tools. In summary, this work can contribute to advancing the theoretical foundations of nonlinear energy system analysis while offering practical tools for engineers working with nonlinear chains, bandpass filters, and periodic signal processing. The results can bridge the gap between abstract functional analysis and applied energy networks design, reinforcing the Volterra series as a powerful tool for modelling energy objects' nonlinear dynamics.

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МОДЕЛЮВАННЯ НЕЛІНІЙНИХ КІЛ ЕНЕГРЕТИЧНИХ СИСТЕМ ЗА ДОПОМОГОЮ РЯДІВ ВОЛЬТЕРРИ: КРИТЕРІЇ ЗБІЖНОСТІ ТА МЕЖІ СТІЙКОСТІ ПРИ ПЕРІОДИЧНОМУ ЗБУДЖЕННІ

У даній роботі отримано достатні умови збіжності рядів Вольтерри, які описують розв'язки класу нелінійних інтегральних рівнянь, що моделюють динамічні кола енергетичних об'єктів із періодичними вхідними сигналами. Шляхом формулювання реакції системи через нелінійне інтегральне рівняння ми встановлюємо суворі критерії абсолютної збіжності розкладу в ряд Вольтерри. Зокрема, досліджено кола енергетичних об'єктів, що містять ідеальні смугові фільтри, збуджені тригонометричними поліноміальними вхідними сигналами – конфігурація, типова для спрощеного аналізу фізично реалізованих систем. Для резистивних нелінійних кіл енергетичних систем показано, що ряд Вольтерри зводиться до степеневого ряду, та наведено явні оцінки його радіусу збіжності (Теоретичне твердження 3). Додатково, Теоретичні твердження 1 та 2 містять узагальнені критерії збіжності, засновані на мінімізації функціоналу в обмеженій просторовій області, що розширює попередні результати для систем типу NARX. Отримані результати можуть сприяти поєднанню теоретичного аналізу з інженерними застосуваннями, пропонуючи практичні інструменти для проектування нелінійних кіл енергетичних систем із передбачуваною динамікою, таких як системи силової електроніки та обробки сигналів.

Ключові слова: *збіжність рядів Вольтерри, нелінійні інтегральні рівняння, динамічні електричні кола, періодичні вхідні сигнали, резистивні нелінійні кола, оцінка радіусу збіжності, апроксимація степеневими рядами, нелінійні енергетичні системи.*

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