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The paper presents a state graph labeling for the generation of test pattern of search tree by user-defined strategy and admissibility characteristics of the evaluate function.

Key words: *state space search, heuristic search, admissibility of the evaluate function.*

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ON THE REPRESENTATION OF NON-LINEAR FUNCTIONS BY FRACTIONAL-POWER SERIES

A method for approximation of relationships by polynomials containing fractional-power terms is proposed, which in many cases makes it possible to cut down the number of computations. The proposed method for representation of relations by fractional-power polynomials features a smaller number of expansion terms while the approximation precision being the same as in the case of the «classical» methods. The method for finding the parameters of such expansions is considered; generalized spline of fractional order (smaller than or equal to unity) is defined. The experimental results on approximation of relations by fractional splines are presented.

Key words: *approximation, non-liner relationships, fractional-power series, splines.*

Introduction. The modeling and identification of ***industrial / manufacturing objects*** requires registration of experimental relationships which are usually represented in discrete form [1]. In this regard, there occurs the problem of analytical representation of the experimental relationships recorded, and in the case when the experiments are conducted in conditions of high data recording noise levels — the problem of fairing the experimental curves. The precision of approximation by the existing interpolation method representation in the form of series [2–6] depends substantially on the distance between the point of approximation and that the formula is referred to. Achieving an acceptable precision of approximation by such methods requires more and more computations as one moves further from the reference point because of the need to process a great number of terms in the expansions of the relationships. So, the development of methods for approximation of complex experimental relationships is of great importance in the modeling and identification of complex processes.

One of the methods that enable one to avoid the approximation precision being worsened as one moves further from the reference point is the *method of approximating relationships by polynomials containing fractional-power terms*.

Let us consider a method of finding the parameters of approximation of this sort and also defining a generalized spline of fractional order smaller than or equal to unity.

Approximation of relationships by fractional splines. The given above definition can be represented by the generalization of the notion of the linear spline. The approximation of experimental relationships by fractional splines may in many cases prove to be more efficient than that by the traditional splines (linear, cubic, etc.), providing a better approximation with the same number of terms of the approximating polynomial (in the case of linear splines) or a comparable approximation precision with a smaller number of terms (in the case of cubic splines).

Let $f(x) \in C([a, b])$. Consider the problem of approximating the function $F(x)$ by a polynomial containing fractional-power terms. Let this function be fixed in some set of points (i.e., let an experimental signal be measured at some moments of time):

$$x_i\colon \{x_j,\ i = 0, 1, \ldots , n\} : a = x_0 < x_1 < \ldots < x_n = b.$$

We will consider the approximation of the given relationship by a polynomial of the following form:

$$\varphi\left(x\right) = \sum_{k=1}^{r} a_k x^{\alpha^k}, r \geq 1. \tag{1}$$

The sought-for parameters in the minimization of the approximation error are not only the coefficients a_k, but also the powers α_k. To find the parameters, let us form the error functional $L(f, \varphi)$. Depending on the type of the problem being solved, various methods can he used to form the functional. The most widespread in practical applications are the uniform (Chebyshev) and mean-square approximations of functions [7–9]. In accordance with these cases, the functional $L(f, \varphi)$ has the following form:

$$L_1\left(f,\varphi\right) = \min_{x\in\left[a,b\right]}\left\{\left(f\left(x\right)-\varphi\left(x\right)\right)^2\right\},$$

$$L_1(f,\varphi)=\int_a^b\left(f(x)-\varphi(x)\right)^2dx.$$

Minimizing the error functional $L(f, \varphi)$ with respect to the parameters a_k and α_k, one can find the sought-for coefficients determining the best approximations of the experimentally recorded function $f(x)$ by a polynomial of form (1).

The advantages of approximation by polynomials of form (1) become more manifest when using sets of polynomials of not very high power on comparatively short intervals (i.e. splines containing fractional-power terms).

The name of the fractional spline $\varphi(x)$: $x \in [a, b]$ approximating on $[a, b]$ the function f: $[a, b]$ will be given to the function φ: $[a, b]$ satisfying the following conditions:

(1) $\varphi(x) \in C([a, b])$;
(2) $\varphi(x_i) = f(x_i)$, $i = 1, n$;
(3) i: $i = 1, n$.

On the interval $[x_i, x_{i+2}]$, $\varphi(x)$ has the following form:

$$\varphi(x)=\varphi_x=a_ix^{\alpha_i}+b_{i,}\,0<\alpha\le 1,$$

$$a_i\in R, b_i\in R.$$

At $\alpha = 1$, the function $\varphi(x)$ is transformed into a linear spline.

Consider an example of finding fractional spline parameters of the approximation of some function $f(x)$ by a polynomial of the form $\varphi(x) = a_1x^{\alpha} + a_2$. Let $f(x)$ be fixed at three points: x_1, x_2, and x_3. Then

$$L(f,\varphi)=\left(f(x_1)-a_1x_1^{\alpha}-a_2\right)^2+\left(f(x_2)-a_1x_2^{\alpha}-a_2\right)^2+\left(f(x_3)-a_1x_3^{\alpha}-a_2\right)^2;$$

$$\frac{\partial L}{\partial a_1}=0;\quad \frac{\partial L}{\partial a_2}=0;\quad \frac{\partial L}{\partial \alpha}=0.$$

The solution of this equation is equivalent to the solution of the following system of nonlinear equations:

$$\begin{cases} f(x_1)-a_1x_1^{\alpha}-a_2=0; \\ f(x_2)-a_1x_2^{\alpha}-a_2=0; \\ f(x_3)-a_1x_3^{\alpha}-a_2=0. \end{cases} \qquad (2)$$

As a result, we get the necessary parameters of approximation of form (1), thus minimizing the approximation error functional. The coefficients of system (2) can be determined by both numerical and analytical methods for solving nonlinear systems.

To study the methods of approximation by generalized fractional splines, a program package was developed in Turbo-Pascal. The functions sinx, e^x and ln$x$ were approximated by polynomials of the form $\varphi(x) = a_1x^{\alpha}+a_2$ on various intervals, and the approximation precision was compared with that achieved with the standard expansions [10] having the same or greater number of terms. The approximation error functional was minimized by the method of descent. The results of numerical experiments are listed in Table 1.

Table 1

The results of numerical experiments on the approximation of relationships by fractional splines

Appr. point	a_1	a_2	α	**Approximation error**	
				standard	**fractional**
$f(x) = \sin x$					
0.8	-0.275	1.002	0.005	0.000032	0.001432
1.4	-0.039	1.002	0.542	0.006313	0.00047
1.7	-0.012	1.0	0.012	0.040245	0.000349
$f(x) = \exp x$					
0.5	1.073	0.981	0.686	0.002477	0.011020
0.8	1.615	0.982	0.975	0.039768	0.003339
1.1	1.747	0.983	1.0	0.297241	0.000214
$f(x) = \ln(1 + x)$					
0.58	-0.678	0.9984	0.4	0.001178	0.001835
0.76	-0.488	1.0	0.422	0.008310	-0.001159
0.85	-0.394	1.0	0.147	0.018719	0.000637

As can be seen, the approximation precision in the case of the standard expansions with a small number of terms decreases as one moves further from the reference point (the zero point for the functions sin$x$, e^x, ln$x$), whereas in the case of fractional-power polynomials the point of approximation has no material effect on the approximation precision.

Conclusions. Thus, the use of fractional-power polynomials and splines is an effective means for approximating complex experimental relationships in scientific research and in the modeling and identification of *manufacturing processes*. In many cases it allows the time taken by computations to be cut down and the problems of modeling complex entities to be solved on a real time basis.

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Запропоновано метод апроксимації функцій поліномами, що містять дрібно-степеневі члени, що у багатьох випадках дозволяє скоротити кількість обчислень. Запропонований спосіб представлення залежностей дрібно-степеневими поліномами характеризується меншим числом членів розкладання, в той час як точність апроксимації є аналогічною, як і у випадку «класичних» методів.

Розглянуто метод знаходження параметрів такого розкладання, визначено узагальнений сплайн дробового порядку (менше або рівного одиниці). Представлені експериментальні результати апроксимації функцій дробовими сплайнами.

Ключові слова: *апроксимація, нелінійні залежності, дрібно-степеневі ряди, сплайни.*

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