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ONE NUMERICAL COMPLEX ANALYSIS METHOD FOR PARAMETERS IDENTIFICATION OF PIECEWISE HOMOGENEOUS CONDUCTIVITY MEDIA WITH USING APPLIED QUASIPOTENTIAL TOMOGRAPHIC DATA

We propose one approach for gradient problems solving of parameters identification in piecewise homogeneous fields based on numerical complex analysis method with using applied quasipotential tomographic data.

Key words: *applied quasipotential tomography, quasiconformal mappings, identification, nonlinear problems.*

Introduction. As it is known (see, for example, [1]), the basic tenets of the electrical impedance tomography (EIT) method were formulated in the last century. Nowadays it is a promising trend that increasingly finds its application in science and technology (see, for example, [1–7]). EIT methods, compared with others, have several advantages [8]. Questions which concern the approach to the selection of a mathematical model of the object, that adequately describes it's properties are considered in the [9]. An iterative method for dividing line zones with different values constant conductions according to the applied potential tomography (APT) is given in [10]. In [11] the forward problem solving exercise by APT proposed modification and the inverse problem — by zones of conductivity; noted that these methods of reconstruction in EIT images can be used to real time mode. An algorithm of the reconstruction of electrical conductivity coefficient (ECC) in doubly-connected region when the inner curve is searching given in [12]. In [5–7] considered the problem of so-called difference EIT, which considers in the reconstruction of ECC when it's dynamics inside the region. The reconstruction of ECC with using Tikhonov's regularizing functional stated in [1, 13–15].

We propose a methodology (like to [13–17]) of ECC identification based on the idea of building the quasiconformal (piecewise-conformal) similarity in the small of a curvilinear quadrangles — dynamical mesh components in the physical domain and the corresponding rectangles in the complex quasipotential domain using applied quasipotential tomographic (AQT) data.

Parameters identification problem of piecewise medias using AQT data. Consider the quasiideal movement processes, for example, electrical charges in the homogeneous curvilinear domain (plate, tomographical cross-section etc) G_z (Fig. 1 a), bounded by limited smooth closed curve

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$\partial G_z = \{(x, y): \quad x = \tilde{x}(\tau), \quad y = \tilde{y}(\tau), \quad 0 \le \tau \le 2\pi, \quad \tilde{x}(0) = \tilde{x}(2\pi) = \tilde{x}_0,$ $\tilde{y}(0) = \tilde{y}(2\pi) = \tilde{y}_0,\ \tilde{x}(\tau),\ \tilde{y}(\tau)$ — *defined continuously differentiated functions*, $O(\tilde{x}_0, \tilde{y}_0)$ — *selected initial reference point*$\}$, generated by the potential difference $\varphi_*^{(p)}$ and $\varphi^{*(p)}$ $\left(\varphi^{*(p)} - \varphi_*^{(p)} > 0\right)$, which are given on a chosen equipotential lines $A_p B_p$ and $C_p D_p$, where A_p, B_p, C_p, D_p — marked points on the ∂G_z; $p = 1, 2, ...$ — some parameter (number of injection [13, 14]); $B_p C_p$ and $A_p D_p$ — impermeable boundary flow lines.

We model the injections of current through the tomographical cross-section with using sets of values $\left\{\tau_A^{(p)}, \tau_B^{(p)}, \tau_C^{(p)}, \tau_D^{(p)}\right\}$, corresponding to which

$$A_p = \left(\tilde{x}\left(\tau_A^{(p)}\right), \tilde{y}\left(\tau_A^{(p)}\right)\right),\ B_p = \left(\tilde{x}\left(\tau_B^{(p)}\right), \tilde{y}\left(\tau_B^{(p)}\right)\right),$$

$$C_p = \left(\tilde{x}\left(\tau_C^{(p)}\right), \tilde{y}\left(\tau_C^{(p)}\right)\right),\ D_p = \left(\tilde{x}\left(\tau_D^{(p)}\right), \tilde{y}\left(\tau_D^{(p)}\right)\right).$$

We denote the corresponding domain bound G_z with given four marked points by $\partial G_z^{(p)}$.

The mathematical physics problem for find the functions $\varphi = \varphi^{(p)}(x, y)$ $\left(\forall p = \overline{1, \tilde{p}}\right)$ having ECC identification conditions $\sigma = \sigma(x, y)$ which is described by us is put in the base of AQT (shunt model) in many cases [13, 14, 18].

$$div\left(\sigma(x, y)\, grad\, \varphi^{(p)}(x, y)\right) = 0, (x, y) \in \tilde{G}_z^{(p)}; \tag{1}$$

$$\varphi^{(p)}\Big|_{A_p B_p} = \varphi_*^{(p)},\ \varphi^{(p)}\Big|_{C_p D_p} = \varphi^{*(p)},$$

$$\sigma \frac{\partial \varphi^{(p)}}{\partial n}\Big|_{B_p C_p} = 0,\ \sigma \frac{\partial \varphi^{(p)}}{\partial n}\Big|_{A_p D_p} = 0; \tag{2}$$

$$\varphi^{(p)}(M)\Big|_{B_p C_p} = \overline{\varphi}^{(p)}(M),\ \varphi^{(p)}(M)\Big|_{A_p D_p} = \underline{\varphi}^{(p)}(M),$$

$$\sigma \frac{\partial \varphi^{(p)}(M)}{\partial n}\Big|_{A_p B_p} = \psi_*^{(p)}(M), \sigma \frac{\partial \varphi^{(p)}(M)}{\partial n}\Big|_{C_p D_p} = \psi^{*(p)}(M). \tag{3}$$

Here $\vec{n}$ — unit vector of outer normal, M — going point of the respectively curve. The functions $\overline{\varphi}^{(p)}(M) = \overline{\varphi}^{(p)}(\tau, ...)$ $\left(\tau_C^{(p)} \le \tau \le \tau_B^{(p)}\right)$,

$\underline{\varphi}^{(p)}(M)=\underline{\varphi}^{(p)}(\tau,\ldots)$ $\left(\tau_A^{(p)}\le\tau\le\tau_D^{(p)}\right)$, $\psi_*^{(p)}(M)=\psi_*^{(p)}(\tau,\ldots)$ $\left(\tau_B^{(p)}\le\tau\le\tau_A^{(p)}\right)$, $\psi^{*(p)}(M)=\psi^{*(p)}(\tau,\ldots)$ $\left(\tau_D^{(p)}\le\tau\le\tau_C^{(p)}\right)$ can be built by the interpolation of the experimentally data $\overline{\varphi}_{\overline{i}^{(p)}}^{(p)}$, $\underline{\varphi}_{\underline{i}^{(p)}}^{(p)}$, $\psi_{*j_*^{(p)}}^{(p)}$, $\psi_{j^{*(p)}}^{*(p)}$ having arguments $\tau=\overline{\tau}_{\overline{i}^{(p)}}^{(p)}$, $\tau=\underline{\tau}_{\underline{i}^{(p)}}^{(p)}$, $\tau=\tau_{*j_*^{(p)}}^{(p)}$, $\tau=\tau_{j^{*(p)}}^{*(p)}$ on the sections B_pC_p, A_pD_p, A_pB_p, C_pD_p respectively $\left(\varphi_*^{(p)}\le\underline{\varphi}_{\underline{i}^{(p)}}^{(p)}\le\varphi^{*(p)},\ \varphi_*^{(p)}\le\overline{\varphi}_{\overline{i}^{(p)}}^{(p)}\le\varphi^{*(p)},\ \psi_{*j_*^{(p)}}^{(p)}>0,\ \psi_{j^{*(p)}}^{*(p)}>0,\ 0\le\overline{i}^{(p)}\le\overline{m}^{*(p)}+1,\ 0\le\underline{i}^{(p)}\le\le\underline{m}_*^{(p)}+1,\ 0\le j_*^{(p)}\le n_*^{(p)}+1,\ 0\le j^{*(p)}\le n^{*(p)}+1\right)$. We search ECC as:

$$\sigma=\left\{\sigma_{k,r}:\underset{\sim}{x}_k\le x\le\underset{\sim}{x}_{k+1},\underset{\sim}{y}_r\le y\le\underset{\sim}{y}_{r+1},k=\overline{0,\underset{\sim}{m}-1},\ r=\overline{0,\underset{\sim}{n}-1},\sigma_{k,r}>0\right\}, \quad (4)$$

where lines $x=\underset{\sim}{x}_k$ and $y=\underset{\sim}{y}_k$ owned the least by the square rectangle area of the domain, the sides of which are parallel to the coordinate axes and which accommodates domains $G_z^{(p)}$; $\tilde{G}_z^{(p)}=G_z^{(p)}\setminus\setminus\left\{z\in G_z^{(p)}:x=\underset{\sim}{x}_k,y=\underset{\sim}{y}_r,k=\overline{1,\underset{\sim}{m}-1},r=\overline{1,\underset{\sim}{n}-1}\right\}$ in itself. Herewith

$$\left[\varphi^{(p)}\right]\Big|_{x=\underset{\sim}{x}_k}=0,\ \left[\varphi^{(p)}\right]\Big|_{y=\underset{\sim}{y}_r}=0,\ \left[\sigma\frac{\partial\varphi^{(p)}}{\partial x}\right]\Bigg|_{x=\underset{\sim}{x}_k}=0,$$

$$\left[\sigma\frac{\partial\varphi^{(p)}}{\partial y}\right]\Bigg|_{y=\underset{\sim}{y}_r}=0,\ \left(k=\overline{1,\underset{\sim}{m}-1},r=\overline{1,\underset{\sim}{n}-1}\right). \quad (5)$$

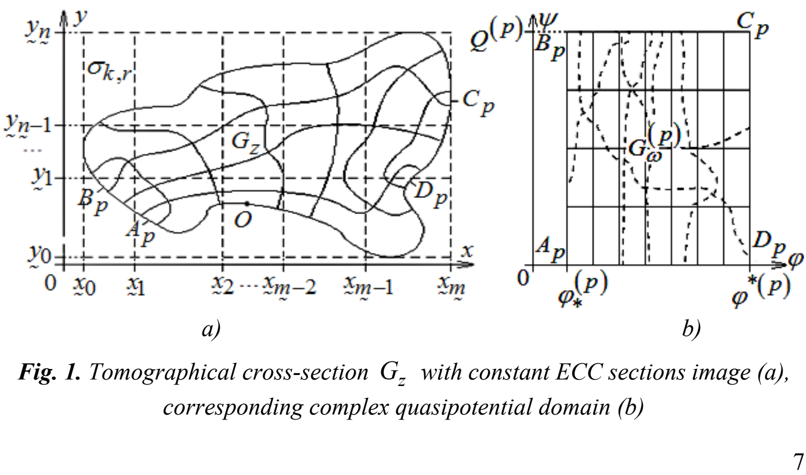

Fig. 1. *Tomographical cross-section G_z with constant ECC sections image (a), corresponding complex quasipotential domain (b)*

We can generalize (1)–(5) to series of more general boundary problems of quasiconformal (piecewise-conformal) mapping of physical domains $G_z^{(p)}$ (Fig. 1 a) onto the corresponding domains of the complex quasipotential $G_\omega^{(p)}$ (Fig. 1 b) (under conditions (4) and (5)) by introducing a flow function $\psi^{(p)} = \psi^{(p)}(x,y)$ complex conjugate to $\varphi^{(p)} = \varphi^{(p)}(x,y)$ $\left(p = \overline{1,\tilde{p}}\right)$ [13, 16, 17, 20]:

$$\sigma\frac{\partial\varphi^{(p)}}{\partial x} = \frac{\partial\psi^{(p)}}{\partial y}, \sigma\frac{\partial\varphi^{(p)}}{\partial y} = -\frac{\partial\psi^{(p)}}{\partial x}; \qquad (6)$$

$$\varphi^{(p)}\Big|_{A_pB_p} = \varphi_*^{(p)}, \varphi^{(p)}\Big|_{C_pD_p} = \varphi^{*(p)}, \psi^{(p)}\Big|_{A_pD_p} = 0,$$

$$\psi^{(p)}\Big|_{B_pC_p} = Q^{(p)}; \qquad (7)$$

$$\int_{MN}\sigma\frac{\partial\varphi^{(p)}}{\partial n}dl = Q^{(p)}, M \in B_pC_p, N \in A_pD_p;$$

$$\varphi^{(p)}\left(M\right)\Big|_{B_pC_p} = \overline{\varphi}^{(p)}\left(M\right),\ \varphi^{(p)}\left(M\right)\Big|_{A_pD_p} = \underline{\varphi}^{(p)}\left(M\right),$$

$$\psi^{(p)}\left(M\right)\Big|_{A_pB_p} = \Psi_*^{(p)}\left(M\right),\ \psi^{(p)}\left(M\right)\Big|_{C_pD_p} = \Psi^{*(p)}\left(M\right), \qquad (8)$$

where

$$G_\omega^{(p)} = \left\{(\varphi,\psi): \varphi_*^{(p)} \le \varphi \le \varphi^{(p)*}, 0 \le \psi \le Q^{(p)}\right\};$$

$$\Psi_*^{(p)}\left(M\right) = \int_{A_pM}\psi_*^{(p)}\left(M\right)dl,$$

$\Psi_*^{(p)}\left(M\right) = \int_{D_pM}\psi_*^{(p)}\left(M\right)dl$; $Q^{(p)}$ — flow of the vector fields (current) through the contact surfaces (A_pB_p and C_pD_p), dl — arc element of corresponding curve.

The synthesis of the numerical quasiconformal mapping method and the rotational block parameterizations ideas. The most popular approach of the solving of the forward problem AQT is based on the using of the finite element method. But this method doesn't satisfy the continuity conditions and, thus, the conservation low on one element of the mesh and whole domain is failed [19].So we use the finite difference method for the discretization of mathematical model's AQT functions and parameters where the electrodynamic analogies principle is considered. And we use quit modified algorithm of the numerical solving of the boundary problems of quasiconformal (piecewise-conformal) mapping of the domains with the different geometric configuration, which are limited by the flow and equiquasipotential lines [16, 20] (including the advantages of the method, described in [16]) for the finding the unknown function of the current's potential $\varphi^{(p)}(x,y)$. We find the functions $\varphi^{(p)}=\varphi^{(p)}(x,y)$ and $\psi^{(p)}=\psi^{(p)}(x,y)$ having a known value of $\sigma=\sigma\left(x,y\right)$ by solving an inverse to (4)–(8) boundary problems of quasiconformal (piecewise-conformal) mapping $G_{\omega}^{(p)}\rightarrow G_{z}^{(p)}$ (relative to $x^{(p)}=x^{(p)}(\varphi,\psi)$ and $y^{(p)}=y^{(p)}(\varphi,\psi)$), statements of that, having condition analogues (5) along the curves $x^{(p)}(\varphi,\psi)=\underset{\sim}{x}_k$, $y^{(p)}(\varphi,\psi)=\underset{\sim}{y}_k$ have the form [13]:

$$\begin{cases}\dfrac{\partial}{\partial\varphi}\left(\dfrac{1}{\sigma}\dfrac{\partial x^{(p)}}{\partial\varphi}\right)+\dfrac{\partial}{\partial\psi}\left(\sigma\dfrac{\partial x^{(p)}}{\partial\psi}\right)=0,\\ \dfrac{\partial}{\partial\varphi}\left(\dfrac{1}{\sigma}\dfrac{\partial y^{(p)}}{\partial\varphi}\right)+\dfrac{\partial}{\partial\psi}\left(\sigma\dfrac{\partial y^{(p)}}{\partial\psi}\right)=0,\ \forall(x^{(p)},y^{(p)})\in G_z^{(p)};\end{cases}\tag{9}$$

$$\begin{gathered}x^{(p)}\left(\varphi_*^{(p)},\psi\right)=\tilde{x}\left(\tau_*^{(p)}\left(\psi\right)\right),\ y^{(p)}\left(\varphi_*^{(p)},\psi\right)=\tilde{y}\left(\tau_*^{(p)}\left(\psi\right)\right),\\ x^{(p)}\left(\varphi,Q^{(p)}\right)=\tilde{x}\left(\overline{\tau}^{(p)}\left(\varphi\right)\right),\ y^{(p)}\left(\varphi,Q^{(p)}\right)=\tilde{y}\left(\overline{\tau}^{(p)}\left(\varphi\right)\right),\\ x^{(p)}\left(\varphi^{*(p)},\psi\right)=\tilde{x}\left(\tau^{*(p)}\left(\psi\right)\right),\ y^{(p)}\left(\varphi^{*(p)},\psi\right)=\tilde{y}\left(\tau^{*(p)}\left(\psi\right)\right),\\ x^{(p)}\left(\varphi,0\right)=\tilde{x}\left(\underline{\tau}^{(p)}\left(\varphi\right)\right),\ y^{(p)}\left(\varphi,0\right)=\tilde{y}\left(\underline{\tau}^{(p)}\left(\varphi\right)\right);\end{gathered}\tag{10}$$

$$\begin{cases}\tilde{x}'(\tau)\dfrac{\partial y^{(p)}\left(\varphi_*^{(p)},\psi_*^{(p)}(\tau,\ldots)\right)}{\partial\varphi}-\tilde{y}'(\tau)\dfrac{\partial x^{(p)}\left(\varphi_*^{(p)},\psi_*^{(p)}(\tau,\ldots)\right)}{\partial\varphi}=\\ =0,\tau\in\left[\tau_B^{(p)};\tau_A^{(p)}\right],\tilde{x}'(\tau)\dfrac{\partial y^{(p)}\left(\varphi^{*(p)},\psi^{*(p)}\left(\tau,\ldots\right)\right)}{\partial\varphi}-\\ -\tilde{y}'(\tau)\dfrac{\partial x^{(p)}\left(\varphi^{*(p)},\psi^{*(p)}\left(\tau,\ldots\right)\right)}{\partial\varphi}=0,\tau\in\left[\tau_D^{(p)};\tau_C^{(p)}\right],\\ \tilde{x}'(\tau)\dfrac{\partial y^{(p)}\left(\underline{\varphi}^{(p)}\left(\tau,\ldots\right),0\right)}{\partial\psi}-\tilde{y}'(\tau)\dfrac{\partial x^{(p)}\left(\underline{\varphi}^{(p)}\left(\tau,\ldots\right),0\right)}{\partial\psi}=0,\\ \tau\in\left[\tau_A^{(p)};\tau_D^{(p)}\right],\tilde{x}'(\tau)\dfrac{\partial y^{(p)}\left(\overline{\varphi}^{(p)}\left(\tau,\ldots\right),Q^{(p)}\right)}{\partial\psi}-\\ -\tilde{y}'(\tau)\dfrac{\partial x^{(p)}\left(\overline{\varphi}^{(p)}\left(\tau,\ldots\right),Q^{(p)}\right)}{\partial\psi}=0,\tau\in\left[\tau_C^{(p)};\tau_B^{(p)}\right],\end{cases}\tag{11}$$

where $\tau=\tau_*^{(p)}(\psi)$, $\tau=\overline{\tau}^{(p)}(\varphi)$, $\tau=\tau^{*(p)}(\psi)$, $\tau=\underline{\tau}^{(p)}(\varphi)$ $\left(\varphi_*^{(p)}\le\varphi\le\varphi^{*(p)},\right.$ $\left.0\le\psi\le Q^{(p)}\right)$ — functions, built by the interpolation of the experimentally data $\tau_{*j_*}^{(p)}$, $\overline{\tau}_{\overline{i}}^{(p)}$, $\tau_{j^*}^{*(p)}$, $\underline{\tau}_{\underline{i}}^{(p)}$ having arguments $\Psi_{*j_*}^{(p)}$, $\overline{\varphi}_{\overline{i}}^{(p)}$, $\Psi_{j^*}^{*(p)}$, $\underline{\varphi}_{\underline{i}}^{(p)}$ on the sections A_pB_p, B_pC_p, C_pD_p, A_pD_p, respectively.

Let's write the difference analogues of (4)–(5), (9)–(11) in mesh domains $G_z^{\gamma(p)}$, similarly to [13], (for the general case of coefficient structure $\sigma(x,y)$) in such form:

$$\begin{cases}\gamma^{(p)2}\left(\sigma_{i,j+1/2}^{\gamma(p)}\left(x_{i,j+1}^{(p)}-x_{i,j}^{(p)}\right)-\sigma_{i,j-1/2}^{\gamma(p)}\left(x_{i,j}^{(p)}-x_{i,j-1}^{(p)}\right)\right)+\\ +\dfrac{x_{i+1,j}^{(p)}-x_{i,j}^{(p)}}{\sigma_{i+1/2,j}^{\gamma(p)}}-\dfrac{x_{i,j}^{(p)}-x_{i-1,j}^{(p)}}{\sigma_{i-1/2,j}^{\gamma(p)}}=0,\gamma^{(p)2}\left(\sigma_{i,j+1/2}^{\gamma(p)}\left(y_{i,j+1}^{(p)}-\right.\right.\\ \left.\left.-y_{i,j}^{(p)}\right)-\sigma_{i,j-1/2}^{\gamma(p)}\left(y_{i,j}^{(p)}-y_{i,j-1}^{(p)}\right)\right)+\dfrac{y_{i+1,j}^{(p)}-y_{i,j}^{(p)}}{\sigma_{i+1/2,j}^{\gamma(p)}}-\\ -\dfrac{y_{i,j}^{(p)}-y_{i-1,j}^{(p)}}{\sigma_{i-1/2,j}^{\gamma(p)}}=0\ \left(1\le i\le m^{(p)},1\le j\le n^{(p)}\right);\end{cases}\quad(12)$$

$$x_{0,j}^{(p)}=\tilde{x}\left(\tau_*^{(p)}\left(\psi_j\right)\right),\ y_{0,j}^{(p)}=\tilde{y}\left(\tau_*^{(p)}\left(\psi_j\right)\right),\ x_{i,n^{(p)}+1}^{(p)}=\tilde{x}\left(\overline{\tau}^{(p)}\left(\varphi_i\right)\right),$$
$$y_{i,n^{(p)}+1}^{(p)}=\tilde{y}\left(\overline{\tau}^{(p)}\left(\varphi_i\right)\right),\ x_{m^{(p)}+1,j}^{(p)}=\tilde{x}\left(\tau^{*(p)}\left(\psi_j\right)\right),$$
$$y_{m^{(p)}+1,j}^{(p)}=\tilde{y}\left(\tau^{*(p)}\left(\psi_j\right)\right),\ x_{i,0}^{(p)}=\tilde{x}\left(\underline{\tau}^{(p)}\left(\varphi_i\right)\right),$$
$$y_{i,0}^{(p)}=\tilde{y}\left(\underline{\tau}^{(p)}\left(\varphi_i\right)\right)\ \left(0\le i\le m^{(p)}+1,0\le j\le n^{(p)}+1\right);\quad(13)$$

$$\begin{cases}\tilde{x}'(\tau)\left(y_{1,j}^{(p)}-y_{0,j}^{(p)}\right)-\tilde{y}'(\tau)\left(x_{1,j}^{(p)}-x_{0,j}^{(p)}\right)=0,\tau\in\left[\tau_B^{(p)};\tau_A^{(p)}\right],\\ \tilde{x}'(\tau)\left(y_{m,j}^{(p)}-x_{m+1,j}^{(p)}\right)-\tilde{y}'(\tau)\left(x_{m,j}^{(p)}-x_{m+1,j}^{(p)}\right)=0,\tau\in\left[\tau_D^{(p)};\tau_C^{(p)}\right],\\ \tilde{x}'(\tau)\left(y_{i,n}^{(p)}-x_{i,n+1}^{(p)}\right)-\tilde{y}'(\tau)\left(x_{i,n}^{(p)}-x_{i,n+1}^{(p)}\right)=0,\tau\in\left[\tau_C^{(p)};\tau_B^{(p)}\right],\\ \tilde{x}'(\tau)\left(y_{i,1}^{(p)}-y_{i,0}^{(p)}\right)-\tilde{y}'(\tau)\left(x_{i,1}^{(p)}-x_{i,0}^{(p)}\right)=0,\tau\in\left[\tau_A^{(p)};\tau_D^{(p)}\right]\\ \left(1\le i\le m^{(p)},1\le j\le n^{(p)}\right),\end{cases}\quad(14)$$

where $\gamma^{(p)}$ — quasiconformal invariant [16] for the corresponding domains $G_\omega^{\gamma(p)}=\left\{\left(\varphi_i^{(p)},\psi_j^{(p)}\right):\quad \varphi_i^{(p)}=\varphi_*^{(p)}+i\Delta\varphi^{(p)},\quad i=\overline{0,m^{(p)}+1};\right.$

$\psi_j^{(p)} = j\Delta\psi^{(p)}, \quad j = \overline{0, n^{(p)}+1}; \quad \Delta\varphi^{(p)} = \dfrac{\varphi^{(p)*} - \varphi_*^{(p)}}{m^{(p)}+1}, \quad \Delta\psi^{(p)} = \dfrac{Q^{(p)}}{n^{(p)}+1},$
$\left.\gamma^{(p)} = \dfrac{\Delta\varphi^{(p)}}{\Delta\psi^{(p)}}, \quad m^{(p)}, n^{(p)} \in \mathrm{N}\right\}; \quad x_{i,j}^{(p)} = x^{(p)}\left(\varphi_i^{(p)}, \psi_j^{(p)}\right), \quad y_{i,j}^{(p)} =$
$= y^{(p)}\left(\varphi_i^{(p)}, \psi_j^{(p)}\right), \quad \sigma_{i,j\pm1/2}^{\gamma(p)} = \sigma\left(\left(x_{i,j\pm1}^{(p)} + x_{i,j}^{(p)}\right)/2, \left(y_{i,j\pm1}^{(p)} + y_{i,j}^{(p)}\right)/2\right),$
$\sigma_{i\pm1/2,j}^{\gamma(p)} = \sigma\left(\left(x_{i\pm1,j}^{(p)} + x_{i,j}^{(p)}\right)/2, \left(y_{i\pm1,j}^{(p)} + y_{i,j}^{(p)}\right)/2\right).$

The general reconstruction algorithm is based on the rotational parameterization of internal nodes of dynamic meshes (which are built for each injections) and the searching ECC. Also, for economy of machine time, we consider, that $\forall k: \underset{\sim}{x}_{k+1} - \underset{\sim}{x}_k = \Delta\underset{\sim}{x}, \ \forall r: \underset{\sim}{y}_{r+1} - \underset{\sim}{y}_r = \Delta\underset{\sim}{y}.$

We organize corresponding to the obtained dynamical mesh «iterative step» for refine ECC $\sigma_{k,r}^{(l)}$ ($l = 0,1,...$ — iterative step number) having fixed $x_{i,j}^{(p,l)}$, $y_{i,j}^{(p,l)}$ as follows: for each $p = \overline{1,\tilde{p}}$, $i = \overline{1,m}$, $j = \overline{1,n}$ we find

$$\sigma_{i+1/2,j+1/2}^{\gamma(p,l)} = \sigma\left(\left(x_{i,j}^{(p,l)} + x_{i+1,j}^{(p,l)} + x_{i,j+1}^{(p,l)} + x_{i+1,j+1}^{(p,l)}\right)/4, \left(y_{i,j}^{(p,l)} + \right.\right.$$
$$\left.\left. + y_{i+1,j}^{(p,l)} + y_{i,j+1}^{(p,l)} + y_{i+1,j+1}^{(p,l)}\right)/4\right)$$

using the idea of quasiconformal (piecewise-conformal) similarity in the small of a curvilinear quadrangles [17]:

$$\sigma_{i+1/2,j+1/2}^{\gamma(p,l)} = \begin{cases} \gamma_{i+1/2,j+1/2}^{(p,l)} / \gamma^{(p)}, \\ if \ \ \sigma_{\min} \le \gamma_{i+1/2,j+1/2}^{(p,l)} / \gamma^{(p)} \le \sigma_{\max}, \\ \sigma_{\min}, \ if \ \ \gamma_{i+1/2,j+1/2}^{(p,l)} / \gamma^{(p)} < \sigma_{\min}, \\ \sigma_{\max}, \ if \ \ \gamma_{i+1/2,j+1/2}^{(p,l)} / \gamma^{(p)} > \sigma_{\max}, \end{cases}$$

where

$$\gamma_{i+1/2,j+1/2}^{(p,l)} = \left(a_{i,j}^{(p,l)} + a_{i,j+1}^{(p,l)}\right) / \left(b_{i,j}^{(p,l)} + b_{i+1,j}^{(p,l)}\right),$$

$$a_{i,j}^{(p,l)} = \left(\left(x_{i+1,j}^{(p,l)} - x_{i,j}^{(p,l)}\right)^2 + \left(y_{i+1,j}^{(p,l)} - y_{i,j}^{(p,l)}\right)^2\right)^{0.5},$$

$$b_{i,j}^{(p,l)} = \left(\left(x_{i,j+1}^{(p,l)} - x_{i,j}^{(p,l)}\right)^2 + \left(y_{i,j+1}^{(p,l)} - y_{i,j}^{(p,l)}\right)^2\right)^{0.5}$$

(we assume values of $\sigma_{\min}$ and $\sigma_{\max}$ as priori known). Then the next ECC approximation in this (l-th) iterative step is

$$\sigma_{k,r}^{(l)} = \frac{1}{\tilde{p}} \sum_{p=1}^{\tilde{p}} \sigma_{k,r}^{(p,l)}, \tag{15}$$

where $\sigma_{k,r}^{(l)}$ — the arithmetical mean value of corresponding $\sigma_{i+1/2,j+1/2}^{\gamma(p,l)}$.

Algorithm for solving the input problem is based on the rotational parameterization of internal nodes of the grid regions $G_z^{\gamma(p)}$, ECC set and using of the block iteration method ideas [21]. We set the number of injections $\tilde{p}$, bound of the domain $G_z^{(p)}$ (using functions $x = \tilde{x}(\tau)$, $y = \tilde{y}(\tau)$), using parameters $\tau_A^{(p)}$, $\tau_B^{(p)}$, $\tau_C^{(p)}$, $\tau_D^{(p)}$ and ε_1, ε_2 (of accuracy), q ($q > 1$ — responsible for the number of iterations of correct of internal nodes having specific ECC), quasipotentials $\varphi_*^{(p)}$, $\varphi^{*(p)}$ and full discharges $Q^{(p)}$, partition parameters $m^{(p)}$, $n^{(p)}$ of the domains $G_\omega^{\gamma(p)}$ (here $\frac{Q^{(p)}}{\varphi^{*(p)} - \varphi_*^{(p)}} \frac{n^{(p)}+1}{m^{(p)}+1} \approx 1$). Then we calculate the coordinates

$$A_p = \left(\tilde{x}\left(\tau_A^{(p)}\right), \tilde{y}\left(\tau_A^{(p)}\right)\right),\ B_p = \left(\tilde{x}\left(\tau_B^{(p)}\right), \tilde{y}\left(\tau_B^{(p)}\right)\right),$$
$$C_p = \left(\tilde{x}\left(\tau_C^{(p)}\right), \tilde{y}\left(\tau_C^{(p)}\right)\right),\ D_p = \left(\tilde{x}\left(\tau_D^{(p)}\right), \tilde{y}\left(\tau_D^{(p)}\right)\right)$$

on the $\partial G_z^{(p)}$, $\Delta\varphi^{(p)} = \left(\varphi^{*(p)} - \varphi_*^{(p)}\right) / \left(m^{(p)} + 1\right)$, $\Delta\psi^{(p)} = Q^{(p)} / \left(n^{(p)} + 1\right)$ and quasiconformal value invariants $\gamma^{(p)} = \Delta\varphi^{(p)} / \Delta\psi^{(p)}$.

Specifying the values of flow $\psi_{*j}^{(p)}$, $\psi_j^{*(p)}$ and potential functions $\overline{\varphi}_i^{(p)}$, $\underline{\varphi}_i^{(p)}$ at the points $\tau_{*j}^{(p)}$, $\tau_j^{*(p)}$, $\overline{\tau}_i^{(p)}$, $\underline{\tau}_i^{(p)}$, respectively, we calculate (13) by interpolation, then we find the coordinates of nodes $x_{0,j}^{(p)}$, $y_{0,j}^{(p)}$, $x_{i,n^{(p)}+1}^{(p)}$, $y_{i,n^{(p)}+1}^{(p)}$, $x_{m^{(p)}+1,j}^{(p)}$, $y_{m^{(p)}+1,j}^{(p)}$, $x_{i,0}^{(p)}$, $y_{i,0}^{(p)}$ $\left(0 \le i \le m^{(p)} + 1, 0 \le j \le n^{(p)} + 1, p = \overline{1, \tilde{p}}\right)$ on the $\partial G_z^{(p)}$. The initial approximations of the $x_{i,j}^{(p,0)}$ and the $y_{i,j}^{(p,0)}$ we obtain, for example, as follows:

$$x_{i,j}^{(p,0)} = \left(x_{i,0}^{(p)} + x_{i,n^{(p)}}^{(p)} + x_{0,j}^{(p)} + x_{m^{(p)},j}^{(p)}\right) / 4,$$
$$y_{i,j}^{(p,0)} = \left(y_{i,0}^{(p)} + y_{i,n^{(p)}}^{(p)} + y_{0,j}^{(p)} + y_{m^{(p)},j}^{(p)}\right) / 4$$
$$\left(1 \le i \le m^{(p)}, 1 \le j \le n^{(p)}, p = \overline{1, \tilde{p}}\right).$$

We set the partition parameters $\underset{\sim}{m}$ and $\underset{\sim}{n}$ (so that their multiplication does not exceed the total number of nodes on the boundaries) and initial approximation of ECC, for example, as follows: $\sigma^{(0)} = \left\{\sigma_{k,r}^{(0)} : \sigma_{k,r}^{(0)} = C, C > 0\right\}$; we set the minimum and maximum possible values for $\forall_{k,r}\sigma_{k,r}^{(l)}$: $\sigma_{\min}$, $\sigma_{\max}$.

We refine the coordinates of near-boundary nodes (using the difference analogs of Cauchy-Riemann conditions) by solving systems of equations

$$\begin{cases} x_{1,j}^{(p,l+1)} = x_{0,j} + 0.5\gamma^{(p)}\sigma_{0,j}^{(l)}\left(y_{0,j+1}^{(p,l)} - y_{0,j-1}^{(p,l)}\right), \\ y_{1,j}^{(p,l+1)} = y_{0,j} - 0.5\gamma^{(p)}\sigma_{0,j}^{(l)}\left(x_{0,j+1}^{(p,l)} - x_{0,j-1}^{(p,l)}\right); \end{cases}$$

$$\begin{cases} x_{m,j}^{(p,l+1)} = x_{m+1,j} - 0.5\gamma^{(p)}\sigma_{m+1,j}^{(l)}\left(y_{m+1,j+1}^{(p,l)} - y_{m+1,j-1}^{(p,l)}\right), \\ y_{m,j}^{(p,l+1)} = y_{m+1,j} + 0.5\gamma^{(p)}\sigma_{m+1,j}^{(l)}\left(x_{m+1,j+1}^{(p,l)} - x_{m+1,j-1}^{(p,l)}\right); \end{cases}$$

$$\begin{cases} x_{i,1}^{(p,l+1)} = x_{i,0} - 0.5\gamma^{(p)}\sigma_{i,0}^{(l)}\left(y_{i+1,0}^{(p,l)} - y_{i-1,0}^{(p,l)}\right), \\ y_{i,1}^{(p,l+1)} = y_{i,0} + 0.5\gamma^{(p)}\sigma_{i,0}^{(l)}\left(x_{i+1,0}^{(p,l)} - x_{i-1,0}^{(p,l)}\right); \end{cases}$$

$$\begin{cases} x_{i,n}^{(p,l+1)} = x_{i,n+1} + 0.5\gamma^{(p)}\sigma_{i,n+1}^{(l)}\left(y_{i+1,n+1}^{(p,l)} - y_{i-1,n+1}^{(p,l)}\right), \\ y_{i,n}^{(p,l+1)} = y_{i,n+1} - 0.5\gamma^{(p)}\sigma_{i,n+1}^{(l)}\left(x_{i+1,n+1}^{(p,l)} - x_{i-1,n+1}^{(p,l)}\right) \end{cases}$$

relative to $\left(x_{1,j}^{(p,l+1)}, y_{1,j}^{(p,l+1)}\right)$, $\left(x_{m,j}^{(p,l+1)}, y_{m,j}^{(p,l+1)}\right)$, $\left(x_{i,1}^{(p,l+1)}, y_{i,1}^{(p,l+1)}\right)$, $\left(x_{i,n}^{(p,l+1)}, y_{i,n}^{(p,l+1)}\right)$, respectively $\left(1 \le i \le m^{(p)},\ 1 \le j \le n^{(p)}\right)$. Further, for finding the coordinates of internal nodes use the formula (12) when $i = \overline{2, m^{(p)} - 1}$, $j = \overline{2, n^{(p)} - 1}$ (here $l = 0,1,...$ — iterative step number) [13, 15, 16]. We repeat this procedure q times, then we form matrix (15).

We have note [16] among the conditions of completion of the iterative process: we need the stabilization of the near-boundary nodes, the ECC, the quasiconformal degree parameter, etc, $\left(1 \le p \le \tilde{p}, 1 \le i \le m, 1 \le j \le n\right)$. In the case where one of these conditions is not satisfied we return to the correction of the coordinates of internal nodes, otherwise — we build the reconstructed image and, — if necessary, — the electrodynamical mesh, the complex quasi-potential domains or calculate the velocity fields by the formula $\vec{v}^{(p)} = \sigma\left(x, y\right) \cdot grad\varphi^{(p)}$, etc.

Note 1. For the providing of iterative process convergence the value of parameter q should be set higher than 1, but note that too large number q is slowing convergence.

Note 2. In this formulation of the problem (as opposed to [16]) there is no necessary to use formulae to refine the boundary nodes (since their coordinates are known a priori).

We represent the results of **numerical calculations** for the following input data: $\tilde{x}(\tau)=150\cos\tau$, $\tilde{y}(\tau)=100\sin\tau$, $m^{(p)}=100$, $q=100$, $\varepsilon_1=10^{-2}$, $\varepsilon_2=10^{-2}$, $\varphi_*^{(p)}=0$, $\varphi^{*(p)}=1$, $\left\{\tau_A^{(p)}=\frac{\pi}{8}+(p-1)\frac{\pi}{40}+\pi,\ \tau_B^{(p)}=\tau_A^{(p)}-\frac{\pi}{4},\ \tau_C^{(p)}=\tau_A^{(p)}-\pi,\ \tau_D^{(p)}=\tau_C^{(p)}-\frac{\pi}{4}\right\}$ $(1\le p\le\tilde{p})$, $\underset{\sim}{m}=100$, $\underset{\sim}{n}\times 75$ $(\Delta\underset{\sim}{x}=3,$ $\Delta\underset{\sim}{y}=2.(6))$, $\sigma_{\min}=1$, $\sigma_{\max}=2$, $\forall_{k,r}\sigma_{k,r}^{(l)}=\sigma_{\min}$ $\left(k=\overline{1,\underset{\sim}{m}-1},r=\overline{1,\underset{\sim}{n}-1}\right)$, $Q^{(p)}$, $\psi_{*j}^{(p)}$, $\psi_j^{*(p)}$ $\overline{\varphi}_i^{(p)}$, $\underline{\varphi}_i^{(p)}$. The obtained results of ECC distribution (Fig. 2 b, Fig. 2 c), where sizes, location and shape of inhomogeneities approximately theoretically correspond to a given (Fig. 2 a).

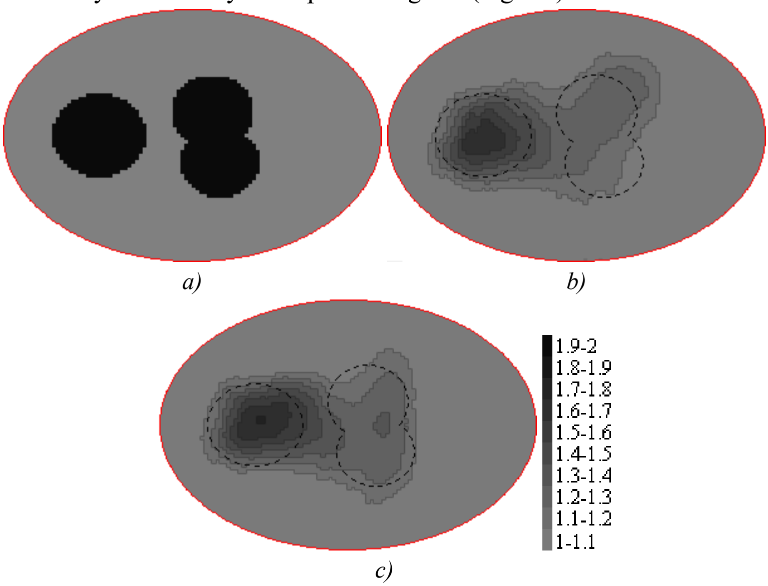

Fig. 2. *ECC distribution: exact solution (a); when* $\tilde{p}=20$ *(b); when* $\tilde{p}=40$ *(c); dashed lines show subregions with ECC equal 2*

Conclusions. We developed the methodology (like as [13–17]) of image reconstruction based on the idea of the quasiconformal (piecewise-conformal) similarity in the small of building a curvilinear quadrangles — dynamical mesh components in the physical domain and the corresponding rectangles in the complex quasipotential domain using AQT data and the rotational parameterization of internal nodes of dynamic meshes (which are built for each injections) and searched ECC. This algorithm is characterized by comparatively high speed of computer convergence (because, unlike from many famous methods, it does not need the searching derivatives of the ECC distribution functions in the determinated points and correction the boundary nodes on each iteration step).

The significant feature of developed algorithm is the possibility it's paralleling and finishing the procedure when quite some conditions of the finishing the process are complete. Also those areas of the physical domain, where the errors of calculation exist are automatically defined. This allows to use the machine time more frugally. The last is essential, particularly, for searching so-named «dead zones» and the «zones of large gradients», which appears near the especial points of unsmooth boundary lines and critical points of insides of respectively domains.

We plan to extend the proposed algorithm to the spatial case and to the several area cases of applied potential of output stream.

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На основі числових методів комплексного аналізу запропоновано підхід до розв'язання градієнтних задач ідентифікації параметрів кусково-однорідних середовищ за даними томографії прикладених квазіпотенціалів.

Ключові слова: *томографія прикладених квазіпотенціалів, квазіконформні відображення, ідентифікація, нелінійні задачі.*

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ПОРІВНЮВАЛЬНА ЕКСПЕРИМЕНТАЛЬНА ОЦІНКА ЕФЕКТИВНОСТІ ЧИСЛОВОЇ РЕАЛІЗАЦІЇ МАТЕМАТИЧНИХ МОДЕЛЕЙ ОБ'ЄКТІВ ІЗ РОЗПОДІЛЕНИМИ ПАРАМЕТРАМИ МЕТОДОМ ОПОРНИХ ПЕРЕРІЗІВ

Шляхом обчислювальних експериментів досліджується ефективність числової реалізації математичних моделей методом опорних перерізів у порівнянні з традиційним широковживаним методом скінчених різниць. Отримані показники затрат процесорного часу при використанні різних методів знаходження розв'язків. Сформульовано рекомендації щодо ефективного використання методу опорних перерізів.

Ключові слова: *ефективність числової реалізації, затрати процесорного часу, різницева схема, метод скінчених різниць.*

Вступ. Існуючі методи і засоби, що використовуються для розв'язування задач моделювання об'єктів з розподіленими параметрами, ґрунтуються в переважній більшості на використанні моделей у вигляді диференціальних рівнянь з частинними похідними [1, 2]. Цей підхід дозволяє забезпечити високий рівень адекватності та ефективне застосування за відсутності специфічних часо-

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