UDC 004.41;004.424;519.64

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# ALGORITHMS FOR DYNAMIC CORRECTION OF THE THERMAL FLOWS' MEASURING SYSTEMS'

Algorithms for thermal flows' measuring systems' dynamic correction are considered. The algorithms are based on the solution of deconvolution problem. The algorithms serve as a basis for development of special-purpose computing means for on-line solution of the dynamics faults' compensation problem in measuring systems.

**Key words:** *measuring systems, dynamic correction, integral model, quaternary algorithm.*

**Introduction.** Traditional approach to solving measuring systems' dynamic correction problem is by using correction circuits in electrical part of the system, and also by the signal restoration problem solving based on the measuring system's differential model [1–3]. The dynamic correction method, described below, is based on the measuring transformer's integral model's inversion by using quaternary algorithms [4; 5].

Sluggish response of thermal radiation detectors due to their non-vanishing heat capacity restricts the performance of systems intended for measuring non-stationary thermal radiation fluxes. The wide range of developed radiant-power detectors [2] on the basis of gradient- type sensors [1] are effectively used for studying processes with time constants of about 20–30 s. Attempts to utilize the readings of these instruments for on-line logging and control of faster processes results in considerable dynamic errors. Methods of compensating for these errors can he elaborated by solving the inverse problem for the equations describing the processes in these devices,

**The problem setup.** The general statement of the problem of establishing the interrelationship between the signal of a thermoelectric gradient detector and the density of the nonstationary thermal radiation flux, incident to the inlet stir- lace involves the solution of the inverse boundary-value problem for the heat conduction equation describing the process in the body of the instrument, Questions concerning the development of algorithms for solving these problems are dealt with in [3], However, the approach involves considerable analytic and algorithmic difficulties since

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real radiant thermal flux detectors have a complex multilayer design and consist of composite materials which are difficult to identify.

**The method and algorithms.** In some cases the approach presented below is more suitable for solving the problem of establishing interrelations between the signal applied to the inlet surface of the detector and the reading of the measuring system. This approach is based on the use of dynamic responses of the instrument transducer. In the investigation of dynamic properties of instrument transducers, extensive use is made of the step response $p(t)$ and the impulse response $g(t)$. The former represents the response of the transducer to an input signal similar to the unit-step function and the latter represents the response to an input signal that is the Dirac delta function. As is known, for stationary linear transducers the relationship between the Input signal $q(t)$ and the output signal $e(t)$ is expressed in the form of the convolution relation

$$e(t)=\int_0^t g(t-\tau)q(\tau)d\tau. \qquad (1)$$

According to [1], a sufficiently simple method of compensating for the dynamic error of a thermal radiation flux measuring system can be suggested that is based on finding the deconvolution $q(t)$, with the impulse response $g(t)$ assumed to be known, i.e. by means of solving Volterra's integral equation of the first kind. This approach seems to be more effective (in terms of the simplicity of the required hardware and software) than the solution of the inverse-heat conduction boundary-value problem described by partial differential equations.

The operating conditions of radiation detectors require the on-line filtering of the input thermal radiation flux incident to the inlet surface of the detector. Therefore it is advisable to design a special compensating computing device (CCD) to solve Volterra’s integral equation of the first kind (1) with respect to the unknown function $q(t)$. The compensating computing device can be connected in series with the detector output of the measuring system (figure 1). It should be noted that the signal filtering problem belongs to the class of ill-posed problems since in the numerical realization of Eq. (1) errors in the measurement of the function $e(t)$ may lead to instability In the resulting solutions [4; 5].

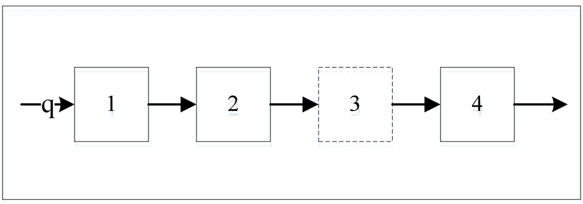


***Fig. 1.*** *Block diagram of thermal-radiation flux measuring system: 1 — thermal radiation sensor, 2 — temperature to electric signal transducer, 3 — compensating computing device, 4 — recording measuring device*

Thus the present article deals with questions concerning the digital simulation of the process of signal filtering in order to develop operation algorithms for special digital computing devices as well as analytical expressions determining the structure of analog devices designed for the same purpose. We consider an example of dynamic-error compensation as a radiant thermal flux detector, which illustrates the suggested digital simulation method.

The impulse response $g(t)$ of the detector is determined from the experimentally found step response by differentiating the latter since $g(t) = dp(t)/dt$. Straightforward measurement of $g(t)$ is not possible due to the difficulties of the practical realization of delta functions.

The function $p(t)$ is found as the response of the detector to an input signal of the form of the unit-step function which is generated, say, by a stationary thermal radiator with an electric filament lamp and an electromagnetically controlled blind.

The experimentally obtained step response $p(t)$ of one of the detectors is plotted in Figure 2. Other types of detectors have similar step responses. To design continuous or sampled-data compensating devices it is necessary to have an analytical expression for the function $g(t)$, which can be obtained by approximating $p_0(t)$ and differentiating this analytical approximation.

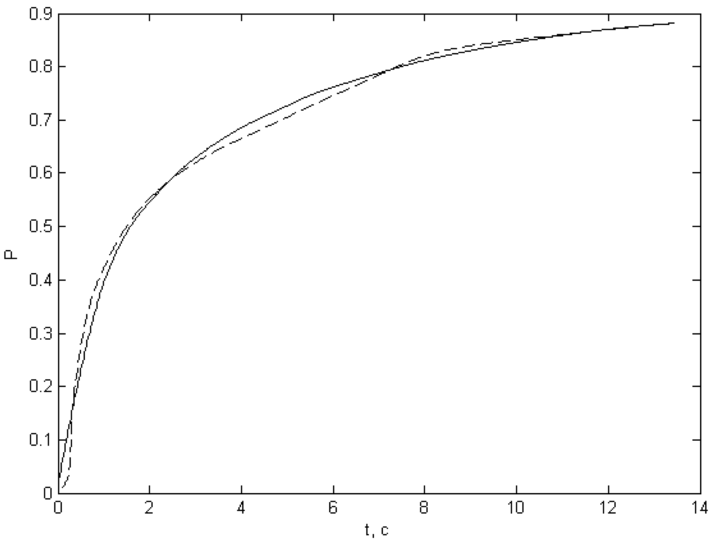


***Fig. 2.*** *Detector step response*

It is often advisable to approximate the response by

$$\tilde{p}(t) = a_0 + e^{-\lambda t} \sum_{i=1}^{M} a_i t^{i-1},\ i = \overline{1, M}, \qquad (2)$$

where $a_0$, $a_i$ and $\lambda$ are constant coefficients [6]. The calculations make it possible to determine the numerical values of the coefficients and the degree $M = 7$. In this case the impulse response of the detector is

$$\tilde{g}(t) = \left(0.562 - 0.376t + 0.102t^2 - 0.01198t^3 + \right.$$
$$\left. + 0.000657t^4 - 0.000016t^5 + 0.000000144t^6\right)e^{-0.178t}, \quad (3)$$

and is shown in Figure 3.

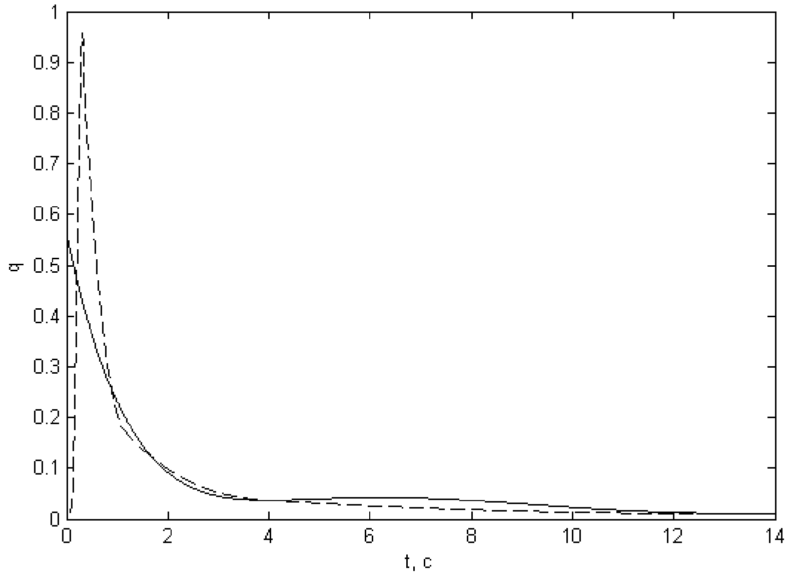


***Fig. 3.*** *Detector impulse response*

Using the expression (2) we can readily find the transfer function of the detector under consideration via the Laplace-Carson transform [6], Implementation of a transfer function with, the aid of operational amplifiers with lags in feedback is relatively straightforward and results in the design of an analog CCD.

In the special case under consideration the CCD is meant to solve the integral equation

$$\int_0^t \left[0.562 - 0.376(t-\tau) + 0.102(t-\tau)^2 - 0.01198(t-\tau)^3 + 0.000657(t-\tau)^4 - \right.$$
$$\left. - 0.000016(t-\tau)^5 + 0.000000144(t-\tau)^6\, e^{-0.178(t-\tau)} q(\tau)\, d\tau = e(t). \quad (4)$$

It is advisable to carry out digital simulation of the integral method for typical operating conditions of the detector. To this end, experiments were carried out to measure the nonstationary thermal radiation flux, which was varied according to a law typical of actual operating conditions of the detectors. The variation of the density of the incident thermal radiation flux was achieved by rotating the detector placed in the radiation field of a stationary thermal radiator about the axis passing through the center of the inlet surface of the detector.

From steady-state measurements it was found that the variation of the radiation flux density corresponds to Lambert's law of cosines with a maximum deviation of 2.5%. The required uniformity of the rotation of the detector in the measurement process was provided by the paper-feed mechanism of the recorder N-37. To exclude the convective component of the thermal flux, the detector with the rotating device was placed in a vacuum. The response data of the receiver to a thermal radiation flux with period $2T_j = 28$ s are presented in Table 1. The resulting function $e_e(t_i)$ defines the operating conditions under consideration and is the right-hand side of Eq. (4).

Table 1

| $N$ | $t_i$ | $e_e(t_i)$ | $N$ | $t_i$ | $e_e(t_i)$ | $N$ | $t$i | $e_e(t_i)$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 0.00 | 11 | 5.00 | 10.20 | 21 | 10.0 | 12,44 |
| 2 | 0.50 | 0.51 | 12 | 5.50 | 10.93 | 22 | 10.5 | 11.78 |
| 3 | 1.00 | 0.79 | 13 | 6.00 | 11.58 | 23 | 11.0 | 11.35 |
| 4 | 1.50 | 2.06 | 14 | 6,50 | 12.23 | 24 | 11.5 | 10.87 |
| 5 | 2.00 | 3.60 | 15 | 7.00 | 12.75 | 25 | 12,0 | 10.22 |
| 6 | 2.50 | 4,76 | 16 | 7.50 | 13.14 | 26 | 12.5 | 9.48 |
| 7 | 3.00 | 5.84 | 17 | 8.00 | 13.30 | 27 | 0.0 | 8.60 |
| 8 | 3.50 | 5.84 | 17 | 8.00 | 13.30 | 27 | 13.0 | 8.60 |
| 9 | 4.00 | 8.01 | 19 | 9,00 | 13.19 | 29 | 14.0 | 6 11 |
| 10 | 4.50 | 9.16 | 20 | 9.50 | 12.89 | | | |

Now we have all the necessary data for the digital implementation of integral equation (1). i.e., the form of kernel (3) and the right hand side. As the numerical method of solving Eq. (4) for digital computer simulation we choose the quadrature formula method according to which the integral is replaced by a finite sum. As a result, we obtain an algebraic system of simultaneous equations:

$$\sum_{j=1}^{i} A_j \tilde{g}\left(t_i - t_j\right) q\left(t_j\right) = e_e\left(t_j\right), i = 1, 2, ..., \tag{5}$$

where $A_j$ are the quadrature formula coefficients, $t_i = (i - 1)h$. and $h$ is the sampling step.

Integral equation (1) is characterized by the property that for associated system (5) it is impossible to determine the value $q(0)$, which is required for subsequent recurrent computation of $q(h)$. $q(2h)$, ... To find

$q(0)$, use can be made of the expression $q(0) = e'_e(0)/g(0)$ [7]. To compute the value

$$e'_e(0) = \left.\frac{de_e(t)}{dt}\right|_{t=0},$$

various interpolation methods are used including the quadrature interpolation formula

$$e'_e \approx \frac{1}{2h}\left[-3e_e(0) + 4e_e(h) - e_e(2h)\right].$$

Now the values

$$\tilde{q}(t_i) = \frac{1}{A_i g(0)}\left(e_e(t_i) - \sum_{j=1}^{i-1} A_j \tilde{g}(t_i - t_j)\tilde{q}(t_j)\right) \quad (6)$$

can he subsequently found from system (5).

Application to Eq, (1) of the trapezoidal rule with a constant step $h$ makes it possible to obtain a recurrence relation of the form

$$\tilde{q}(0) = \frac{e'_e(0)}{\tilde{g}(0)},$$
$$\tilde{q}(t_i) = \frac{2}{A_i g(0)}\left(\frac{e_e(t_i)}{h} - \sum_{j=1}^{i-1} A_j \tilde{g}(t_i - t_j)\tilde{q}(t_j)\right), \quad (7)$$

where

$$A_j = \begin{cases} 0.5 \; as \; j = 1; \\ 1 \quad as \; j > 1. \end{cases}$$

It is seen from expression (7) that each step the number of operations to be performed increases with the index; accordingly, the storage needed for computer simulation increases. To overcome this difficulty an approach is suggested in [8] which is based on the successive translation of the origin to the point $t_v$, where $t_v = t = 0$, In this case a change in the initial time results in a change in the initial condition, i.e., for $t_v$ the system has a new initial condition. However this approach does not yield effective results for the hardware implementation of integral equation (1), which is due to the necessity of performing a number of operations to find new initial conditions for the points $t_v$, $t_{v+1}$, $t_{v+2}$,... Therefore in the case under consideration it is advisable to use a modified algorithm for the numerical solution of integral equation (1); it is based on the separability of the kernel [9]. In this case the kernel of Eq. (1) is represented as

$$g(t-\tau) = \sum_{i=1}^{m} \alpha_i(t)\beta_i(\tau), \; i = \overline{1,m}. \quad (8)$$

This representation of the kernel makes it possible to write Eq. (4) in a conveniently solvable form:

$$\Big[B_1R_1+B_2\left(tR_1-R_2\right)+B_3\left(t^2R_1-2tR_2+R_3\right)+$$
$$+B_4\left(t^3R_1-3t^2R_2-3tR_3-R_4\right)+B_5\left(t^4R_1+\right.$$
$$\left.+4t^3R_2+6t^2R_3-4tR_4+R_5\right)+B_6\left(t^5R_1-5t^4R_2+\right.$$
$$\left.+10t^3R_3-10t^2R_4+5tR_5-R_6\right)+B_7\left(t^6R_1-6t^5R_2+15t^4R_3-\right.$$
$$\left.-20t^3R_4+15t^2R_5-6tR_6-R_7\right)\Big]e^{-0178t}=e\left(t\right),$$

where

$$B_1=0.562;\ B_2=-0.376;\ B_3=0.102;$$
$$B_4=-0.01198;\ B_5=0.000657;$$
$$B_5=-0.000016;\ B_7=0.000000144;$$
$$R_1=\int_0^t e^{0.178\tau}q\left(\tau\right)d\tau;\ R_2=\int_0^t e^{0.178\tau}\tau q\left(\tau\right)d\tau;$$
$$R_3=\int_0^t e^{0.178\tau}\tau^2q\left(\tau\right)d\tau;\ R_4=\int_0^t e^{0.178\tau}\tau^3q\left(\tau\right)d\tau;$$
$$R_5=\int_0^t e^{0.178\tau}\tau^4q\left(\tau\right)d\tau;\ R_6=\int_0^t e^{0.178\tau}\tau^5q\left(\tau\right)d\tau;$$
$$R_7=\int_0^t e^{0.178\tau}\tau^6q\left(\tau\right)d\tau;$$

In the case of kernel (8), expression (7) becomes

$$\tilde{q}\left(0\right)=\frac{e_e'\left(0\right)}{\sum_{i=1}^{m}\alpha_i\left(0\right)\beta_i\left(0\right)},$$
$$\tilde{q}\left(t_i\right)=\frac{2}{A_i\Theta}\left(\frac{e_e\left(t_i\right)}{h}-\sum_{i=1}^{m}\alpha_i\left(t_i\right)\sum_{j=1}^{i-1}A_j\beta_i\left(t_j\right)q\left(t_j\right)\right),\tag{9}$$

where

$$\Theta=\sum_{i=1}^{m}\alpha_i\left(t_i\right)\beta_i\left(t_i\right)=\tilde{g}\left(t_i-t_i\right)=\tilde{g}\left(0\right).$$

From (9) the conclusion can be drawn that the number of operations to be performed does not depend on the index of iteration since the sumands $\sum_{j=1}^{i-1} A_j \beta_i \left(t_j\right) q\left(t_j\right)$ depend only on one independent variable, $t_j$. It should be noted that we can associate with the quadrature formula method a regularization algorithm in which the regularization parameter is the step of the quadrature |10j. In the present work the step is found experimentally proceeding from the condition $\max\left|q\left(t\right)-\tilde{q}\left(t\right)\right| = \min,\ t \in \left[0; Tf\right],$ where q($t$) is the exact solution of the benchmark problem corresponding to the standard op rating conditions of the measuring system; the quantity $Tf$ is equal to the half-period of the input sine waveform and is 14 s.

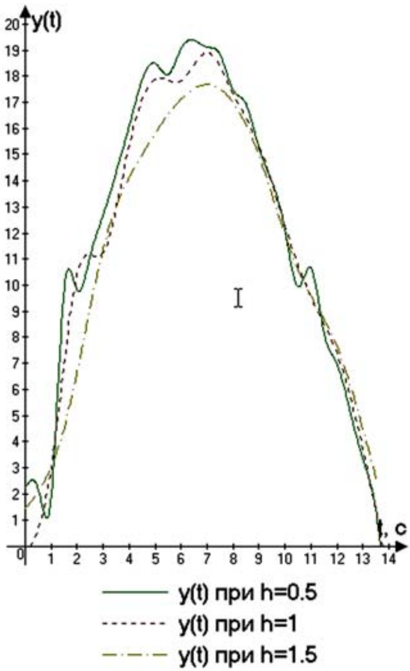


***Fig. 4.*** *Plot of input signal obtained by deconvolution*

In Figure 4 the filtered input signal $q$ is presented for various values of the step $h$. It should be noted that the stability of the resulting solution is improved as step is increased, which demonstrates the regularization effect of the latter.

The Accuracy and the stability of the solution of integral equation (l) can be improved by improving the accuracy of the kernel approximation. Thus it is advisable to apply the method of cubic splines (piecewise polynomial functions) which provide not only the required approximation accuracy but also the continuity of the derivatives at the interpolation points [11]. In this case the responses $p_e(t)$ and $g(t)$ of the radiation detector (Figure 3) me approximated be corresponding polynomials of the form

$$\tilde{p}_e(t) = A_z(T_z - t)^3 + B_z(t - T_{z-1})^3 + S_z(T_z - t) + D_z(t - T_{z-1}); \quad (10)$$

and

$$\tilde{g}_e(t) = A_z^*(T_z - t)^2 + B_z^*(t - T_{z-1})^2 + S_z^* + D_z^*, \quad (11)$$

where $z = 2, 3, \ldots, n$; $T_{z-1} \le t \le T_z$; $A_z$, $B_z$, $S_z$, $D_z$, $A_z^*, B_z^*, S_z^*, D_z^*$ and $T_z$ are constant coefficients.

To solve integral equation (1) with a kernel defined by (11) numerically it is advisable to apply mean-rectangle formulas because for $t = 0$ the function (11) has the value $\tilde{g}_e(0) = 0$ and it is difficult to apply the trapezoidal rule. However, the approximation of the step response of the detector by polynomials of form (2) does not permit taking into account the values of the function $p_e(t)$ on the interval [0; 0.25]. A numerical study has shown that for detectors of the class under consideration the process taking place on the interval [0; 0.25] does not affect the solution of Eq. (1) significantly. The recurrence relation based on the application of the mean rectangle formula and the separability property of kernel (11) is

$$\tilde{q}\left(\frac{h}{2}\right) = \frac{e_e(h)}{\tilde{g}_e\left(\frac{h}{2}\right)},$$

$$\tilde{q}\left(i_{i-\frac{1}{2}}\right) = \frac{1}{\tilde{g}_e\left(\frac{h}{2}\right)}\left(\frac{e_e(t_i)}{h} - F\right), \quad (12)$$

where

$$F = A_z^*\left((T_z - t)^2 C_1 + 2(T_z - t)C_2 + C_3\right) +$$
$$+B_z^*\left((t - T_{z-1})^2 C_1 - 2(t - T_{z-1})C_2 + C_3\right) +$$
$$+\left(S_z^* + D_z^*\right)C_1; \; C_1 = \sum_{j=0}^{i-2} \tilde{q}\left(t_{j+\frac{1}{2}}\right);$$
$$C_2 = \sum_{j=0}^{i-2} t_{j+\frac{1}{2}} \tilde{q}\left(t_{j+\frac{1}{2}}\right); \; C_3 = \sum_{j=0}^{i-2} t^2_{j+\frac{1}{2}} \tilde{q}\left(t_{j+\frac{1}{2}}\right);$$
$$t_{j+\frac{1}{2}} = \left(j + \frac{1}{2}\right)h; \quad z = 2, 3, \ldots, n.$$

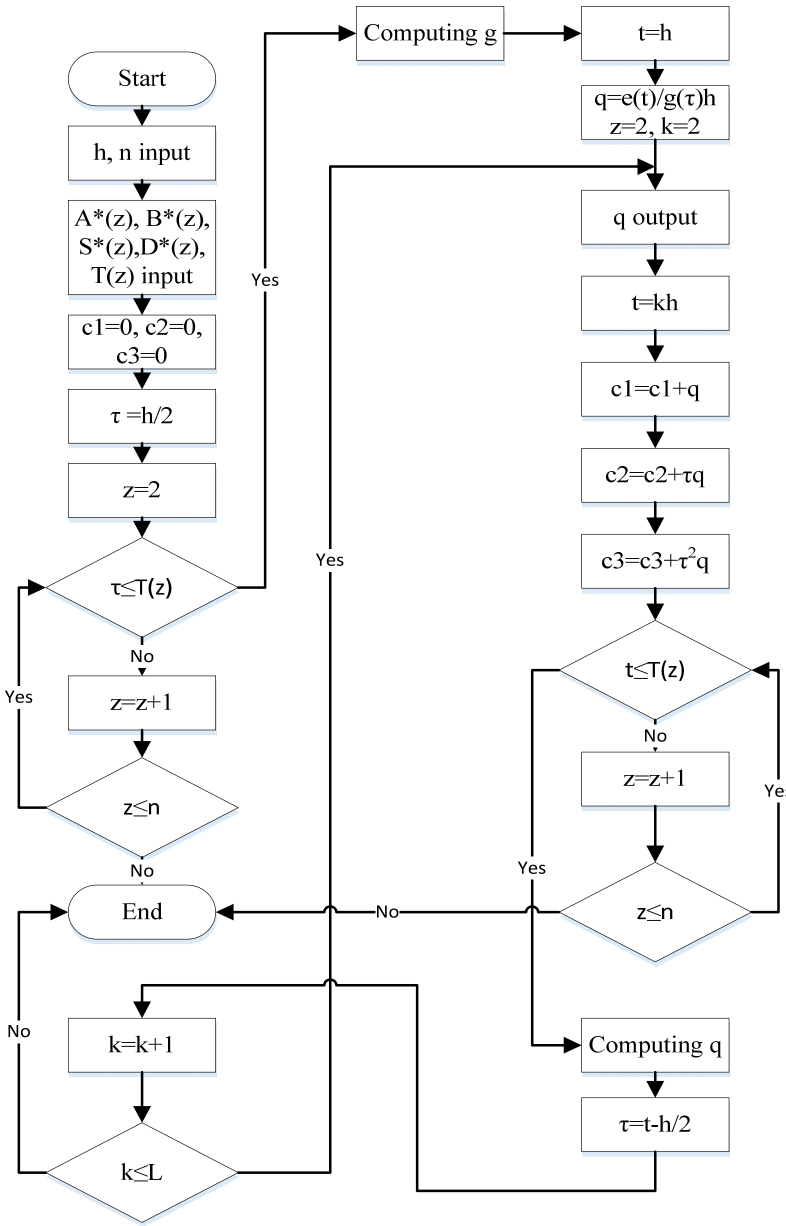


***Fig. 5.*** *Flow chart of algorithm for solving Volterra's integral equation of the first kind.*

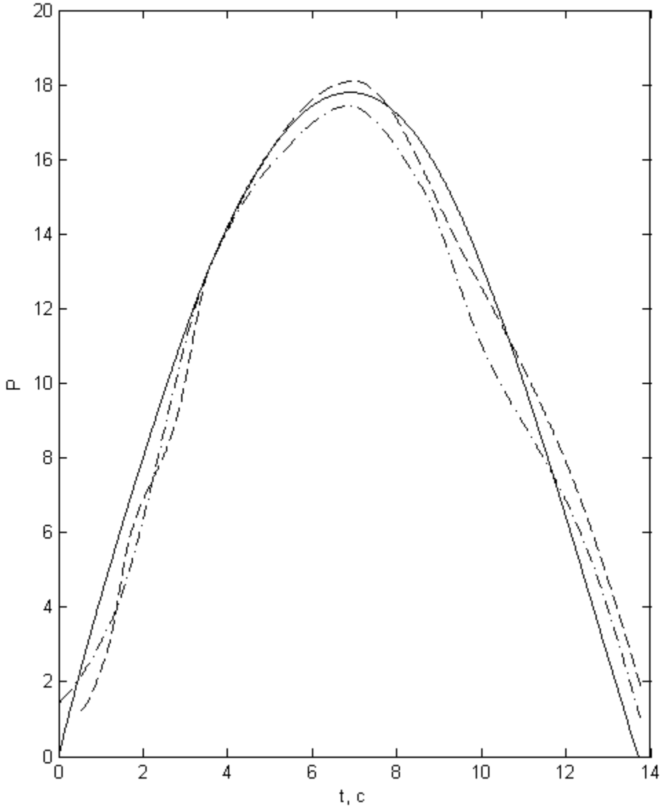


***Fig. 6.*** *Computed and ideal input signals*

Results. The flowchart of the resulting algorithm (12) is shown in Figure 5. The numerical experiments were conducted using mathematical modeling software MATLAB and Simulink. The results of the deconvolution of the sinusoidal wave form input by applying the algorithm (7) for step $h = 1.5$ and by applying the algorithm (12) for step $h = 0.5$ are represented by graphs in Figure 6 (curves 1 and 2 respectively), where the ideal signal is also shown (curve 3). The results obtained imply that for the detector class under consideration the maximum deviation of the values of the filtered signal from the real signal does not exceed 11%, which indicates the practical applicability of the method.

**Conclusion.** Thus we have elaborated and investigated an integral method of dynamic error compensation in a thermal radiation flux measuring system by means of a digital compensation device and a basic approximating expression (4) tor analog-device design**.**

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Розглядаються алгоритми динамічної корекції систем вимірювання теплових потоків. Алгоритми основані на вирішенні задачі зворотної згортки. Вони служать основою для розробки спеціалізованих комп'ютерних засобів для вирішення в реальному часі задачі компенсації динамічних похибок у вимірювальних системах.

**Ключові слова:** *вимірювальні системи, динамічна корекція, інтегральна модель, квадратурний алгоритм.*

Отримано: 25.08.2016